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SPECIAL ISSUE

# ANALYTICAL SOLUTION FOR TWO-DIMENSIONAL SOIL-STRUCTURE INTERACTION WITH NORMAL PLANE P-WAVE INCIDENCE

VASKO KOKALANOV*, VLADO GICEV, NATASHA STOJKOVIKJ

*Faculty of Computer Science, Goce Delcev University, Krste Misirkov, 10A, 2000 Stip, North Macedonia*

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ABSTRACT: In this paper an analytical solution for two dimensional Soil-Structure Interaction (2D SSI) model is presented. The model consists of three parts: stress-free elastic, isotropic, homogeneous ground half-space, idealized rigid semicircular foundation and rectangular homogeneous building. The interactive system is excited by vertical incident P-wave. The presented analytical solution satisfies the stress-free boundary conditions at the half-space surface. The analyzed solutions for foundation motion and the relative response of the building can be further used for verification of numerical models developed for 2D SSI

KEY WORDS: Soil-stricture interaction, wave propagation, P-wave.

## 1 INTRODUCTION

The serious consequences to the structures that some earthquakes (Niigita 1964, Mexico City 1985, Kobe 1995) has taught the engineers that the structures are not independent systems. Many of the damages and lost lives could have been saved if they were treated as interactive system with the soil. Studying of the interaction of the three constitutive elements of the soil-foundation-structure system is particularly important for design and retrofit of important structures as nuclear power plants, long bridges, tall buildings, etc. which collapse would cause tremendous disasters. The fear of possible damage on nuclear plants, the Engineering Earthquake Research Center in the University of Berkley was fully focused on study the soil structure interaction (SSI) effects controlling nuclear power plant seismic response [1]. First SSI models were based on homogenous half space with surface rigid stamp [1]. The expansion of informatics and development of the computer capacities has helped many scientific fields to expand the margins of the problems that they are dealing with. Among the branches that benefit from the computer era is the numerical calculus [2].

*Corresponding author e-mail: vasko.kokalanov@ugd.edu.mk

None of the numerical models is applicable if it has not been verified by analytical solution previously. Using the procedure of the analytical model for SSI under incident SH waves, developed by Luco [3], for vertical incidence only and by Trifunac [4] for arbitrary incidence, this paper shows the derivation of the analytical SSI model under action of P-SV incident waves.

## 2 Soil Structure Interaction

### 2.1 Soil motion

The equation that is going to illustrate the soil-structure interaction is derived based on a model containing infinitely long elastic shear wall of height $H$ and thickness $h$ founded on rigid foundation with semicircular section which is embedded in elastic, isotropic and homogeneous soil. The soil is subjected to incident plane wave, traveling through the soil in vertical direction

$$u^{(i)} = Ae^{-i(k_\alpha z+\omega t)}\,, \tag{1}$$

where $k_\alpha$ is the wave number for the P-wave, $k_\alpha = \omega/\alpha$, $\omega$ is the frequency of the wave, $\alpha$, $\beta$ is the velocities of the P- and SV-wave respectively, $A$ is the amplitude of incident plane wave, $i$ is the imagination number and $t$ marks the time.

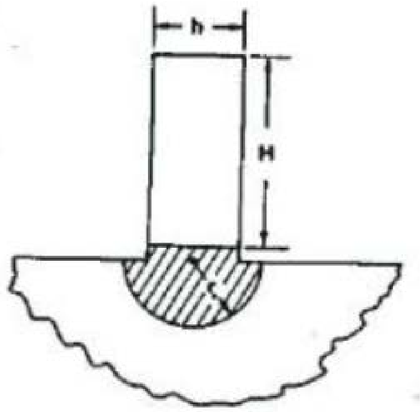


Fig. 1. Shear wall, foundation and soil [3].

The total displacement from the incident P-wave consist of the free field, which is sum of the displacements of the incident wave, $u^{(i)}$ and the reflected wave $u^{(ir)}$ without presence of the foundation, and the dissipated wave produced from foundation's vibration, $u^{(fr)}$ [5].

$$u_z = u^{(i)} + u^{(ir)} + u^{(fr)}\,. \tag{2}$$

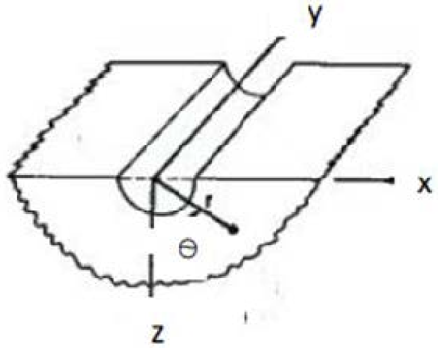


Fig. 2. The Deckard and cylindrical coordinates.

The total displacement must satisfy the wave equation at each soil point ($r \geq h/2$ and $|\theta| \leq \pi/2$)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}=\frac{1}{\alpha}\frac{\partial^2 u_z}{\partial t^2}\,. \tag{3}$$

The total displacement must satisfy the boundary conditions of wave equation in cylindrical coordinates at each points of the free surface ($r \geq h/2$ and $|\theta| = \pi/2$, Fig. 2):

$$\sigma_{\theta z} = -\frac{\mu_s}{r}\frac{\partial u_z}{\partial \theta} = 0\,, \tag{4}$$

where $\mu_s$ is Lame parameter of the soil.

At the soil-foundation interface points ($r = h/2$ and $|\theta| \leq \pi/2$), the conditions of continuity of displacements must be satisfied as well

$$u^{(i)} = \Delta e^{-i\omega t}\,, \tag{5}$$

where $\Delta$ is the unknown amplitude that should be determined with this model.

The free field displacement expressed with (6) below can be modified using the Euler's relations [8] between the exponential and trigonometric functions (7)

$$u^{(ff)} = u^{(i)} + u^{(ir)} = Ae^{-i\omega t}(e^{-ik_\alpha z} + e^{ik_\alpha z})\,, \tag{6}$$

$$\cos\theta = \frac{1}{2}(e^{-i\theta} + e^{i\theta})\,. \tag{7}$$

Knowing that $z = r\cos\theta$ and substituting (6) in (7), we get

$$\left(e^{-ik_\alpha z} + e^{ik_\alpha z}\right) = 2\cos(k_\alpha r\cos\theta)\,. \tag{8}$$

Expanding (8) in Jacobi-Anger series [6], we obtain

$$\cos\left(y\cos\theta\right) = J_0 + 2\sum_{n=1}^{\infty}\left(-1\right)^n J_{2n}\left(y\right)\cos(2n\theta)\,. \tag{9}$$

According to this, one can express the free field displacement with the Bessel functions of first kind [6].

$$u^{(ff)} = Ae^{-i\omega t}\left[2J_0(k_\alpha r) + 4\sum_{n=1}^{\infty}\left(-1\right)^n J_{2n}\left(k_\alpha r\right)\cos\left(2n\theta\right)\right]. \tag{10}$$

The foundation is obstacle in the path of the incident wave. Hence, according to physics of waves, when the wave hits the foundation, one part of the wave should be reflected and one refracted. Since the foundation is ideally stiff the refracted part of the wave does not exist. The reflected wave is traveling to the infinity and hence, besides the wave equation, it must satisfy the Sommerfeld's radiation conditions at infinity [7]. That involves the Henkel functions in the description of the soil displacements from the reflected wave

$$u^{(fr)} = e^{-i\omega t}\left[a_0 H_0^{(1)}(k_\alpha r) + \sum_{n=1}^{\infty} a_n H_{2n}^{(1)}\left(k_\alpha r\right)\cos\left(2n\theta\right)\right]. \tag{11}$$

The coefficient $a_0$ and $a_n$ are obtained by imposing (5)

$$a_0 = \left[\Delta - 2J_0\left(k_\alpha r\right)A\right]\frac{1}{H_0^{(1)}(k_\alpha r)}\,, \tag{12}$$

$$a_n = (-1)^{n+1}\frac{4J_{2n}(k_\alpha r)}{H_{2n}^{(1)}(k_\alpha r)}A, \quad n = 1, 2, 3\ldots \tag{13}$$

In total three forces are influencing the foundation. The first one $f_z^b$ is vertical and it is coming from the weight of the wall. The other two, are from the influence of the soil on the foundation in horizontal ($f_x^s$) and vertical direction ($f_z^s$). The total displacement is a result of state of foundation's equilibrium of these three forces and the inertial force

$$-\omega^2 M_0 \Delta e^{-i\omega t} = \left(f_z^s + f_z^b\right), \qquad -\omega^2 M_0 \Delta e^{-i\omega t} = f_x^s\,, \tag{14}$$

where $M_0$ is the mass of foundation.

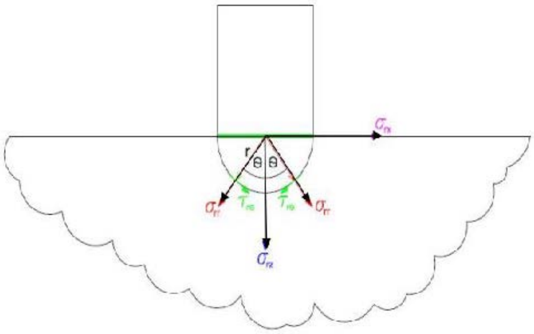

Fig. 3. Radial stresses in cylindrical coordinate system.

The force from the soil that acts horizontally on the foundation can be determined by integration of the horizontal projection of the stresses at the soil-foundation interface

$$f_x^s = \int_{-\pi/2}^{\pi/2} \sigma_{rx} a^2 \sin\theta d\theta\,, \qquad f_z^s = 2\int_{0}^{\pi/2} \sigma_{rz} a^2 \sin\theta d\theta\,. \tag{15}$$

Figure 3 illustrates that the stresses $\sigma_{rx}$ and $\sigma_{rz}$ are obtained as horizontal and vertical projection of $\sigma_{rr}$ and $\tau_{r\theta}$ stresses (16). According to the angular coordinate $\theta$, one can conclude that one half of the horizontally projected stresses are in negative and the other half are in positive direction. Hence, in horizontal direction there are two equal forces acting in opposite direction which leads to resulting force equal to 0. On the left half $\sigma_{rr}$ is in negative direction $\tau_{r\theta}$ is in poisitive direction, and on the right half $\sigma_{rr}$ is in positive direction and $\tau_{r\theta}$ is in negative direction. On the other hand, for the vertically projected stresses on both halves they have same signs and hence the symmetry is used and the integration is made only for half of the interval

$$\begin{bmatrix}\sigma_{rx}\\ \sigma_{rz}\end{bmatrix} = \begin{bmatrix}-\sin\theta & \cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}\begin{bmatrix}\sigma_{rr}\\ \tau_{r\theta}\end{bmatrix}, \qquad \begin{bmatrix}\sigma_{rx}\\ \sigma_{rz}\end{bmatrix} = \begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}\begin{bmatrix}\sigma_{rr}\\ \tau_{r\theta}\end{bmatrix}. \tag{16}$$

Substitution of the relations (16) into (15), leads to the expressions of the forces of the horizontal and the vertical influence of the soil to the foundation

$$f_x^s = \int_{-\frac{\pi}{2}}^{0} (-\sigma_{rr}\sin\theta + \tau_{r\theta}\cos\theta) r d\theta - \int_{0}^{\frac{\pi}{2}} (\sigma_{rr}\sin\theta - \tau_{r\theta}\cos\theta) r d\theta = 0\,, \tag{17}$$

$$f_z^s = 2\int_{0}^{\pi/2} (\sigma_{rr}\cos\theta + \tau_{r\theta}\sin\theta) r d\theta\,, \tag{18}$$

where

$$\text{(19)}\qquad \begin{aligned} &\sigma_{rr} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{\partial u_\theta}{2\partial\theta} + \frac{u_r}{r} + \frac{\partial u_y}{\partial r}\right) + 2\mu\left(\frac{\partial u_r}{\partial r}\right)\,, \\ &\tau_{r\theta} = \mu\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{\partial u_r}{r\partial\theta}\right)\,, \\ &u_r = u_z\cos\theta\,, \qquad u_\theta = u_z sin\theta\,. \end{aligned}$$

The total displacement in $z$-direction is obtained substituting (10) and (11) in (2)

$$\text{(20)}\quad u_z = e^{-i\omega t}\Bigg(A\Big[2J_0(k_\alpha r) + 4\sum_{n=1}^{\infty}(-1)^n J_{2n}(k_\alpha r)\cos(2n\theta)\Big] + \Big[a_0H_0^{(1)}(k_\alpha r) + \sum_{n=1}^{\infty} a_n H_{2n}^{(1)}(k_\alpha r)\cos(2n\theta)\Big]\Bigg).$$

Substituting solutions for $a_0$ (12) and $a_n$ (13) into (20) leads to

$$\text{(21)}\quad \Delta u_z = e^{-i\omega t}\Bigg(A\Big[2J_0(k_\alpha r) + \sum_{n=1}^{\infty}(-1)^n J_{2n}(k_\alpha r)\cos(2n\theta)\Big] + \Big[-2J_0(k_\alpha r)A + 4\sum_{n=1}^{\infty}(-1)^{n+1}J_{2n}(k_\alpha r)A\cos(2n\theta)\Big]\Bigg)$$

$$\text{(22)}\quad \frac{\partial u_r}{\partial r} = \frac{\partial u_z}{\partial r}\cos\theta = e^{-i\omega t}\Big[-\Delta\frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)} + 2A\frac{J_0'(k_\alpha r)H_0^{(1)}(k_\alpha r) - J_0(k_\alpha r)H_0^{'(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)}\Big]\cos\theta\,.$$

Analyzing the numerator of the second term in the summation

$$\text{(23)}\quad J_0'(k_\alpha r)H_0^{(1)}(k_\alpha r) - J_0(k_\alpha r)H_0^{'(1)}(k_\alpha r) = -J_1'(k_\alpha r)H_0^{(1)}(k_\alpha r) - J_0(k_\alpha r)(-1)H_1^{'(1)}(k_\alpha r)$$

and implementing the cross product of the Bessel functions [9]

$$\text{(24)}\qquad J_{n+1}'(x)\,H_n^{(1)}(x) - J_n(x)\,H_{n+1}^{'(1)}(x) = \frac{2i}{\pi x}$$

leads to the final form of the partial derivative $\dfrac{\partial u_r}{\partial r}$

$$(25)\qquad \frac{\partial u_r}{\partial r} = e^{-i\omega t}\left[-\Delta\frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)} - A\frac{4i}{\pi k_\alpha r H_0^{(1)}(k_\alpha r)}\right]\cos\theta\,.$$

According to the derivation of $\dfrac{\delta u_r}{\delta r}$, the remaining three partial derivatives of term (19) are derived

$$(26)\qquad \frac{\partial u_r}{\partial \theta} = \frac{\partial u_z}{\partial \theta}\cos\theta = u_z\frac{\partial}{\partial\theta}\cos\theta = -u_z\sin\theta\,,$$

$$(27)\qquad \frac{\partial u_\theta}{\partial r} = \frac{\partial u_z}{\partial r}\sin\theta = e^{-i\omega t}\left[-\Delta\frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)} - A\frac{4i}{\pi k_\alpha r H_0^{(1)}(k_\alpha r)}\right]\sin\theta\,,$$

$$(28)\qquad \frac{\partial u_\theta}{\partial \theta} = \frac{\partial u_z}{\partial \theta}\sin\theta = u_z\frac{\partial}{\partial\theta}\sin\theta = u_z\cos\theta\,.$$

Substituting the last four expressions in (19) will give the final terms for calculation of the normal stress in radial direction and the shear stress $\sigma_{rr}$ and $\tau_{r\theta}$ involved in (18). Because the displacement in z-direction is expressed with Bessel functions that have argument different than the variable which is integrated, the integral in (18) is solved by use of the tabular integrals of trigonometric functions (29)

$$(29)\qquad \int_0^{\pi/2}\cos^2\theta d\theta = \frac{\pi}{4}; \qquad \int_0^{\pi/2}\sin^2\theta d\theta = \frac{\pi}{4}\,.$$

After the integration, the force of influence from the soil to the foundation is obtained.

$$(30)\qquad f_z^s = -\frac{M_s k_\alpha e^{-i\omega t} w^2}{r_0}\left(\frac{1}{k_\alpha^2} + \frac{1}{k_\beta^2}\right)\left[-\Delta\frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)} - \frac{4Ai}{\pi k_\alpha r H_0^{(1)}(k_\alpha r)}\right] + \frac{2M_s\omega^2}{r_0^2}\left(\frac{3}{k_\beta^2} - \frac{1}{k_\alpha^2}\right)\Delta e^{-i\omega t}$$

where $k_\alpha = \dfrac{\omega}{\alpha}$, $k_\beta = \dfrac{\omega}{\beta}$ are the wave numbers, $\alpha$ and $\beta$ are the velocities of the P- and SV-wave respectively and $M_s = \dfrac{r_0^2\pi}{2}\rho_s$ is the mass of the soil per unit length removed for the foundation.

### 2.2 Influence of the structure to the foundation

The influence force from the wall to the foundation is calculated as product of the normal stress and the contact area

$$
\begin{aligned}
f_z^b &= 2r\sigma_Z^Z = 2rE_0 \frac{\partial u_Z}{\partial Z}\Big|_{Z=H} \\
&= 2r\Delta\bar{k}_\alpha \tan\left(\bar{k}_\alpha H\right) = \omega^2 M_b \left(\frac{\tan\left(k_\alpha^b H\right)}{k_\alpha^b H}\right) \Delta e^{-i\omega t}\,,
\end{aligned} \tag{31}
$$

where $\bar{k}_\alpha = \dfrac{\omega}{\alpha_0} = \dfrac{\omega}{\sqrt{E_0/\rho_0}}$, $M_b = 2r_0 H\rho_b$, $E_0$ is the elasticity modulus of the soil and $\rho_0$ is soil's density.

### 2.3 Interaction

Substituting the forces form (30) and (31) into the equation of equilibrium (14), will result in to equation with one unknown $\Delta$, the desired displacement (22)

$$
\begin{aligned}
-\omega^2 M_0 \Delta e^{-i\omega t} &= -\frac{M_s k_\alpha e^{-i\omega t}\omega^2}{r_0} \\
&\times \left(\frac{1}{k_\alpha^2} + \frac{1}{k_\beta^2}\right)\left[-\Delta\frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)} - \frac{4Ai}{\pi k_\alpha r H_0^{(1)}(k_\alpha r)}\right] \\
&+ \frac{2M_s\omega^2}{r_0^2}\left(\frac{3}{k_\beta^2} - \frac{1}{k_\alpha^2}\right)\Delta e^{-i\omega t} + \omega^2 M_b\left(\frac{\tan(k_\alpha^b H)}{k_\alpha^b H}\right)\Delta e^{-i\omega t}\,,
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
\Delta\Big[\frac{M_0}{M_s} + \frac{2}{r_0^2}\Big(\frac{3}{k_\beta^2} - \frac{1}{k_\alpha^2}\Big) + \frac{M_b}{M_s}\Big(\frac{\tan(k_\alpha^b H)}{k_\alpha^b H}\Big) - \frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)}\Big(\frac{1}{k_\beta^2} + \frac{1}{k_\alpha^2}\Big)\Big] \\
= \frac{4Ai}{\pi r_0^2 H_0^{(1)}(k_\alpha r)}\left(\frac{1}{k_\beta^2} + \frac{1}{k_\alpha^2}\right).
\end{aligned} \tag{33}
$$

Because the incoming P-wave and the reflected wave, which is also P-wave, are in same phase, there is amplification of the amplitude. Recalling the assumption that the foundation is ideally rigid and all its points are displaced equally, there is need that the displacement which is analyzed must be normalized with twice the amplitude of the P-wave.

$$
\frac{\Delta}{2A} = \Omega\,, \tag{34}
$$

$$(35)\quad \frac{1}{\Omega} = \frac{i\pi r_0^2 H_0^{(1)}(k_\alpha r)}{2}\frac{k_\alpha^2 k_\beta^2}{k_\alpha^2 + k_\beta^2}\left[\frac{M_0}{M_s} + \frac{2}{r_0^2}\left(\frac{3}{k_\beta^2} - \frac{1}{k_\alpha^2}\right)\right.$$
$$\left. + \frac{M_b}{M_s}\left(\frac{\tan\left(k_\alpha^b H\right)}{k_\alpha^b H}\right) - \frac{H_1^{(1)}(k_\alpha r)}{H_0^{(1)}(k_\alpha r)}\left(\frac{1}{k_\beta^2} + \frac{1}{k_\alpha^2}\right)\right].$$

## 3 Analysis

By changing the material properties, the masses and the densities, or the geometries, the expression (34) gives opportunity to obtain information of behavior of various combinations of structures, foundations and soils.

The diagram in Fig. 4 is showing the results from arbitrary chosen values given in Table 1 for the variable parameters of the equation (35). The abscissa axis demonstrates the incident wave's spectra through non-dimensional frequency $k_\alpha r_0$ and the ordinates represents the displacement $\Omega$ obtained with (35).

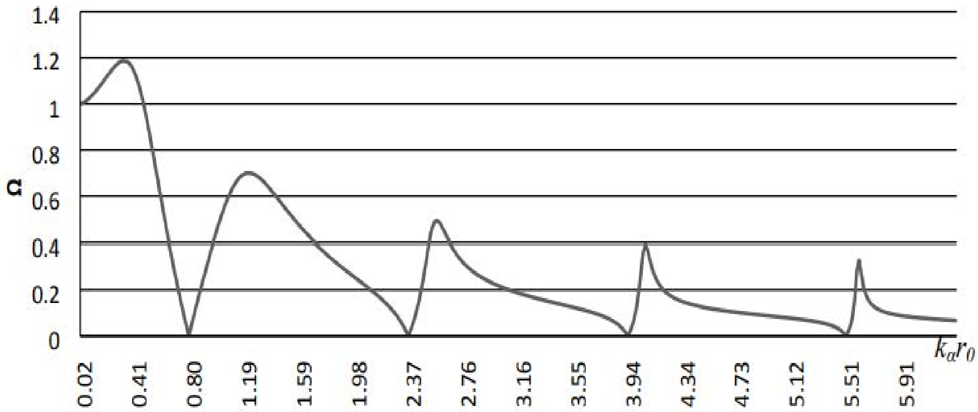


Fig. 4. Analytical solution for parameter from Table 1.

Table 1. Parameters for computation of the analytical solution

| | | | | |
|---|---|---|---|---|
| $H=16$ m | $\alpha_s=346.41$ m/s | $\beta_s=200$ m/s | $\rho_s=1000$ kg/m$^3$ | $M_s=100530.9649$ kg |
| $r_0=8.0$ m | $\alpha_b=346.41$ m/s | $\beta_s=200$ m/s | $\rho_b=1000$ kg/m$^3$ | $M_b=201061.9298$ kg |
| $A=0.5$ m | $\alpha_f=1732.05$ m/s | $\beta_f=1000$ m/s | $\rho_f=1000$ kg/m$^3$ | $M_0=100530.9649$ kg |

In case of incident wave with frequency 0 the system is loaded statically and hence the displacement at the beginning of the graph line is $\Omega = 1$. The statically loading is the part of (35) which is outside of the braces and it tends to be 1 when $k_\alpha r_0$ tends to be 0. This term doesn't depend on the masses of the foundation and the object which

(a)

(b)

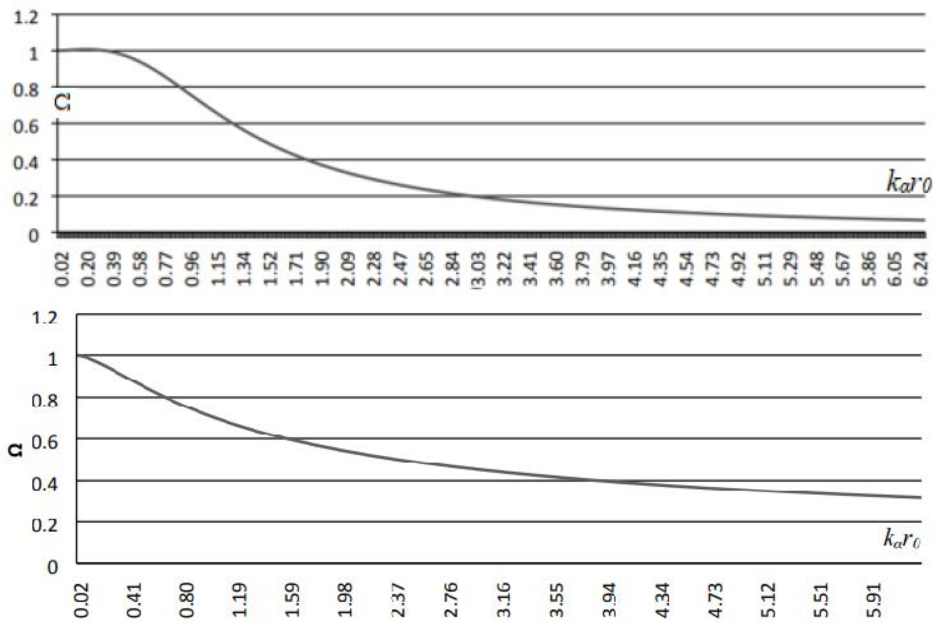


Fig. 5. Analytical solution for half-space.

is proven with graph of solution when there is no object – halfspace (Fig. 5a) and case of halfspace with circular opened channel (Fig. 5b) valda. The parameter values for these two graphs are given in Tables 2 and 3, respectively.

Table 2. Parameters for analytical solution for half-space

| | | | | |
|---|---|---|---|---|
| $H=0$ m | $\alpha_s=346.41$ m/s | $\beta_s=200$ m/s | $\rho_s=1000$ kg/m$^3$ | $M_s=100530.9649$ kg |
| $r_0=8.0$ m | $\alpha_b=346.41$ m/s | $\beta_b=200$ m/s | $\rho_b=1000$ kg/m$^3$ | $M_b=0$ kg |
| $A=0.5$ m | $\alpha_f=346.41$ m/s | $\beta_f=200$ m/s | $\rho_f=1000$ kg/m$^3$ | $M_0=100530.9649$ kg |

Table 3. Parameters for analytical solution for half-space with presence of circular opened channel

| | | | | |
|---|---|---|---|---|
| $H=0$ m | $\alpha_s=346.41$ m/s | $\beta_s=200$ m/s | $\rho_s=1000$ kg/m$^3$ | $M_s=100530.9649$ kg |
| $r_0=8.0$ m | $\alpha_b=346.41$ m/s | $\beta_b=200$ m/s | $\rho_b=1000$ kg/m$^3$ | $M_b=0$ kg |
| $A=0.5$ m | $\alpha_f=346.41$ m/s | $\beta_f=200$ m/s | $\rho_f=1000$ kg/m$^3$ | $M_0=0$ kg |

From Fig. 4 one can notice that with increase of $k_\alpha r_0$ the displacement is raising before it suddenly drops to 0. The maximum value of the in this range of the graph is the maximum displacement that can occur in the foundation. This maximum, and the

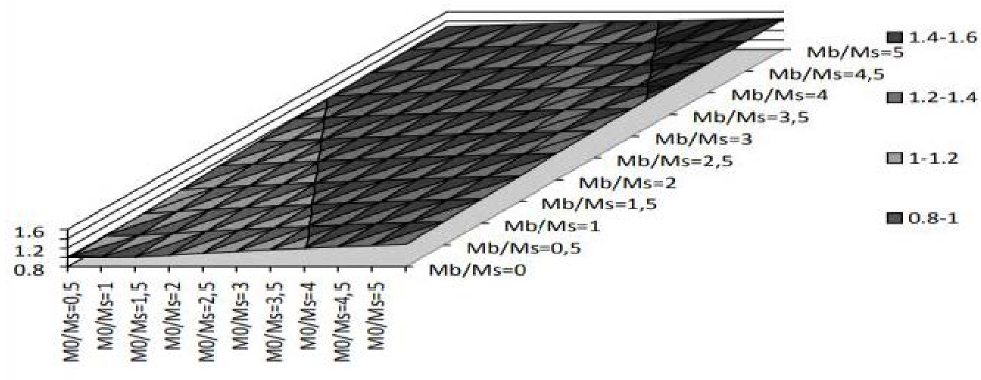


Fig. 6. Change of the amplitude's maximum depending on mass ration.

frequency of the incident wave which makes this displacement, are dependent on the foundation's and structure's stiffness represented by the masses of the foundation and the structure respectively. Figure 6 is showing that with incensement of the stiffness the maximum value is increased. The lowest value is when all masses are equal to one, and the greatest value is when the mass of the structure, $M_b$, is 4.5 times greater than the mass of the soil, $M_s$, and the mass of the foundation, $M_0$, is 5 times greater than mass of the soil, $M_s$.

Despite the influence on the maximum values, with change of the stiffness one changes also the frequency of the incident wave which generates the maximal displacement. The five curves in Fig. 7 are obtained for five different structure stiffness ($M_b/M_0 = 2^k$, where $k = \{-2, -1, 0, 1, 2\}$). This graph is showing that for more

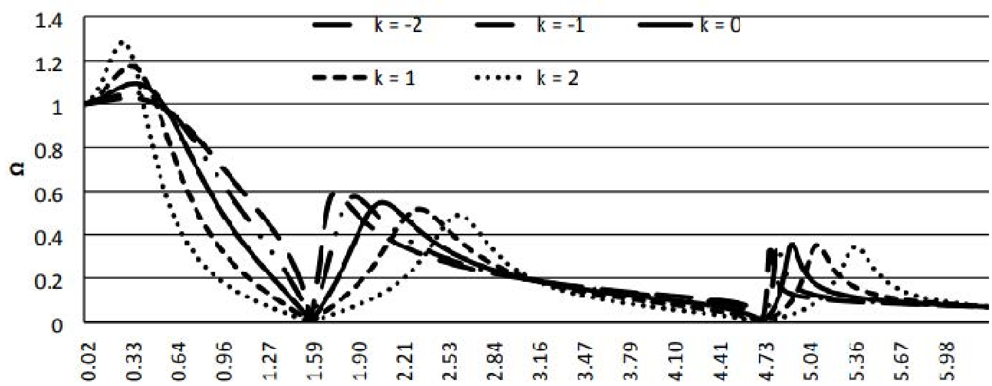


Fig. 7. Influence of the stiffness on the maximal displacements.

stiff structure incident waves with lower frequencies are producing grater displacements. Contrast to these, the maximal values between the zeros are bigger for less stiff structures. Opposite to the previous rules for the maximal displacement, the maximums between the zeros are appearing in higher frequencies for structures with greater stiffness. Each next maximal displacement is smaller than the previous one, which indicates that waves with larger frequencies are incite smaller displacements, no matter of the stiffness of the structure.

From (35) one can see that if the natural frequency of the structure is taken as input, then this term results in 0. Knowing this, we can recognize that the natural frequencies of the structure are the zeros in graphs. For example for the structure described with Table.4 the natural frequencies are $(2n+1)\pi/4$, where $n = 0, 1, 2, 3 \ldots$

## 4 Relative Response

The amplitudes of the relative oscillations on top of the building are of great interest because they can destroy the building if the building is exceeding the linear response range. In [4] the relative response is defined as

$$|r_u| = \Big| |u_z|_{\bar{z}=0} - \Delta \Big| = |\Delta| \left| \frac{1}{\cos \bar{k} H} - 1 \right| ,$$

where $\bar{k} = \omega / \sqrt{E_b/\rho_b}$ is the wave number at the building and $D$ is the displacement calculated with (33).

To include the relation of the geometrical and the material properties of the building and the halfspace we have used dimensionless ratio $\varepsilon = \dfrac{\bar{k}H}{k_\beta a}$.

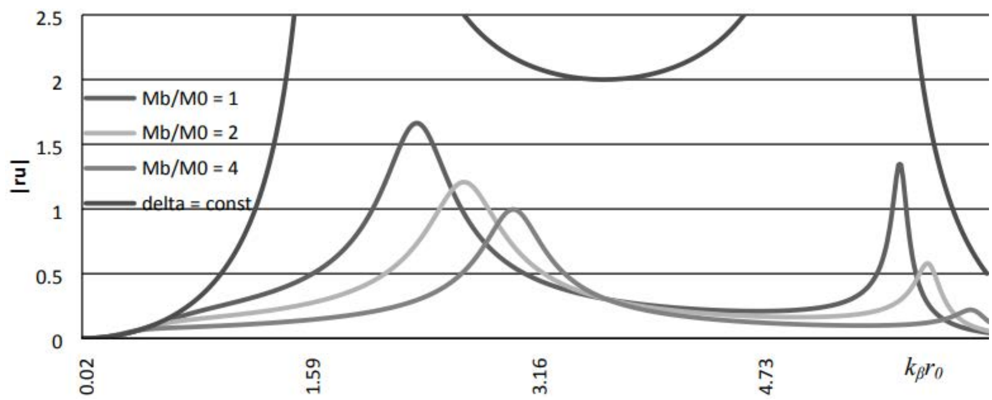


Fig. 8. Relative displacement for $M_0/M_s = 1$; $\varepsilon = 2$; $M_b/M_s = 1, 2, 4$ and $D = 0.5$.

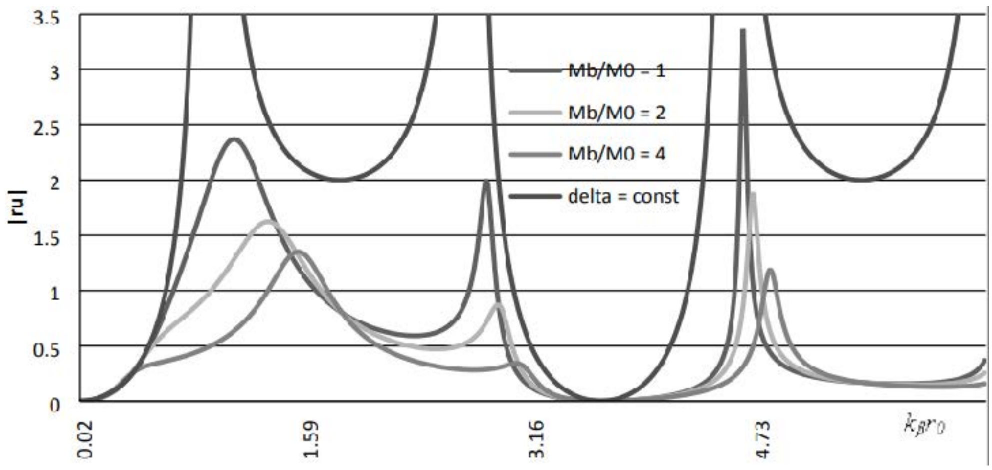


Fig. 9. Relative displacement for $M_0/M_s = 1$; $\varepsilon = 4$; $M_b/M_s = 1, 2, 4$ and $D = 0.5$.

In both figures, Fig. 8 and Fig. 9, the purple lines that are tending to infinity are plots of the curves of $|D| = 0.5$. The point where they tend to infinity is the natural frequency of the building $\bar{k}H = (2n + 1)\frac{\pi}{2}$, $n = 0, 1, 2, \ldots$ The other three curves are finite because $D = 0$ at natural frequencies and infinite value multiplied with zero results in a finite number. This shows that the soil-structure interaction effect is functioning as a “damper” of the building response [10].

Another conclusion can be extracted from the diagrams in Fig. 8 and Fig. 9. When the ratio $M_b/M_s$ is increasing, the amplitudes of the peak value are declining.

Regarding $\varepsilon$, one can conclude that the increase of this dimensionless parameter increases also the corresponding amplitudes, and the curves of the relative response are narrowly banded around the natural frequencies.

## 5 Conclusions

In this paper an analytical solution for 2D SSI model of building on semicircular foundation subjected to incidence P-wave is presented. The model is based on three main assumptions:

- The shear wall is infinitely long and elastic because only by increasing the length of the wall in perpendicular direction of the analyzed plane can make the wall almost infinitely rigid in that direction and hence the deformation of interest are those analyzed in the paper.
- The foundation is rigid with semicircular section because the influence of the foundation on the elastic soil is dependent on and is changing with the distance from the center of the interaction layer between the foundation and the wall.

- The soil is elastic, isotropic and homogeneous soil which is ideal for deriving analytical solution of the problem analyzed in the paper but is rare case in the real world.

This can be a starting point for further analysis with different assumptions or less approximations.

Set of parameters $M_b/M_s$, $M_f/M_s$ and $\varepsilon = \dfrac{\bar{k}H}{k_\beta r_0}$ are used for description and plotting the effects of the interaction and the relative response of the building. The effects of the interaction are depending on the frequency of the incidence wave and affect the amplitude of the foundation's response.

- If building's material (stiffness) and geometrical (height) properties are satisfying $\bar{k}H = (2n+1)\frac{\pi}{2}$, $n = 0, 1, 2, \ldots$, the building is at its natural frequencies and the amplitude of the foundation's displacement is tending to zero.

- In case of incident wave with frequency 0 the system is loaded statically and hence the displacement is equal to one.

- This maximum amplitude of the foundations displacement, and the frequency of the incident wave which makes this displacement, are dependent on the foundation's and structure's stiffness represented by the masses of the foundation and the structure respectively. For more stiff structures the maximum amplitudes are greater.

- For more stiff structure the incident waves with lower frequencies are producing grater displacements.

The relative response is the difference between the displacement at the bottom and the top of the building

- If the interaction is excluded, the relative response tends to infinite at the natural frequencies of the structure.

- The soil-structure interaction effect is functioning as a "damper" of the building response.

- When the ratio $M_b/M_s$ is increasing, the amplitudes of the peak value of the relative response are declining.

- The increase of increases also the corresponding amplitudes and the curves of the $\varepsilon = \dfrac{\bar{k}H}{k_\beta r_0}$ relative response are narrowly banded around the natural frequencies.

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