Journal of Theoretical and Applied Mechanics, Sofia, Vol.55 (2025) pp. 35-51
SPECIAL ISSUE

MODELING HUMANOID ROBOT MOUTH EXPRESSIONS REFLECTING EMOTIONAL STATES VIA ELLIPTIC INTUITIONISTIC FUZZY SETS

VELICHKA TRANEVA[1*], STOYAN TRANEV[1],
VENELIN TODODOROV[2,3]

[1]*Prof. Asen Zlatarov University, Prof. Yakimov Blvd., Bourgas 8000, Bulgaria*
[2]*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev Str., Bl. 8, Sofia 1113, Bulgaria*
Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, 25A Acad. Georgi Bonchev Str., 1113 Sofia, Bulgaria

[Received: 26 December 2024. Accepted: 12 March 2025]

doi: https://doi.org/10.55787/jtams.25.55.1.035

ABSTRACT: This paper introduces a novel approach to modeling humanoid robot mouth expressions using Elliptic Intuitionistic Fuzzy Sets (E-IFS). The E-IFS model employs elliptic intuitionistic fuzzy logic to capture the complexity of human emotions, enabling robots to express nuanced emotional states through mouth movements. By effectively handling uncertainties and emotional intensity variations, the model offers a flexible and adaptive alternative to traditional techniques. Results demonstrate the model's capability to generate realistic, contextually appropriate expressions, enhancing human-robot interaction. These findings highlight its potential for making robots more emotionally expressive. Future research will focus on expanding the model for multi-modal emotional expression and assessing its impact on user perception.

KEY WORDS: Elliptic Intuitionistic Fuzzy Sets, Humanoid Robot, Index Matrix, Mimics.

1 INTRODUCTION

A humanoid robot (HR) is a robot designed to mimic human behavior by having a body shape similar to that of a person [1]. Humanoid robots often express emotions through their mouths, making this a primary channel for conveying emotional states. The literature has explored various applications of humanoid robots, including space operations (Tzvetkova, 2014 [2]), handling and positioning work objects (Sharari, 2015 [3]), bipedal locomotion control (Tawara et al., 2001 [4]), cooperative object manipulation (Hawley and Suleiman, 2019 [5]), gender representation in robotics (Carpenter et al., 2009 [6]), emotional and sociable humanoid robots

*Corresponding author e-mail: veleka13@gmail.com

(Breazeal, 2003 [7]), health assistance (Robins et al., 2005 [8]), customer acceptance (Belanche et al., 2020 [9]), and speaker recognition (Ding and Shi, 2017 [10]). In the rapidly evolving field of robotics, the development of emotive humanoid robots is becoming increasingly important to enhance human-robot interaction. Emotion-expressive humanoid robots are more adept at communication and are more likely to be accepted in a wide range of social and service-oriented roles. One of the most crucial facial features for conveying a variety of emotions, such as surprise, happiness, grief, and anger, is the mouth. Therefore, accurate and nuanced mouth expression modeling is essential for creating realistic and expressive emotional representations in humanoid robots. Traditional methods for generating facial expressions in robots often rely on predefined templates or simple geometric transformations. Although these approaches can produce basic expressions, they often fail to capture the subtle nuances and variations inherent in human emotions. Moreover, such methods may lack the flexibility to accommodate the ambiguous and imprecise nature of emotional expression, which can vary significantly across individuals and contexts.

To address these challenges, this study introduces a novel approach to modeling humanoid robot mouth expressions using Elliptic Intuitionistic Fuzzy Sets (E-IFS) [11]. Each element in the E-IFS is surrounded by an ellipse, incorporating different levels of membership and non-membership. Fuzzy sets, particularly E-IFS, are well-suited for emotion modeling due to their ability to handle imprecision and uncertainty. By leveraging the mathematical properties of E-IFS, the proposed method effectively represents a range of possible mouth shapes corresponding to different emotional states. By integrating E-IFS logic, the model provides a more comprehensive and flexible framework for emotional representation by accounting for degrees of hesitation, membership, and non-membership, which fluctuate within an elliptical region associated with each expression. The main contributions of this study are as follows: Firstly, we present a comprehensive procedure for generating E-IFS-based mouth expressions, demonstrating how altering elliptic parameters can reflect various emotional states. Second, we implement our approach on a humanoid robot and demonstrate its effectiveness in producing realistic and situation-appropriate expressions.

In Section 2, we review the relevant literature on fuzzy set theory and its extensions for emotional expression in humanoid robots. Section 3 introduces the fundamental definitions of E-IF matrices (E-IFIMs) and E-IF quads (E-IFQs). Section 4 provides a detailed explanation of the proposed E-IFS model, while Section 5 outlines the experimental setup used to validate our methodology. This study aims to advance research in robotic emotional expression and contribute to ongoing efforts to make humanoid robots more relatable and intuitive for human users. Section 6 examines the feasibility of the proposed E-IFS approach for simulating humanoid robot mouth expressions. Finally, Section 7 presents a summary of the study and suggests directions for future research.

2 Literature Review

This section provides an extensive assessment of the literature on **recent advancements and theoretical developments** related to **fuzzy set models**. Humanoid robots, designed to **mimic human features and capabilities**, are becoming increasingly **integrated into human society** to enhance the overall **quality of life**. Most human activities are inherently emotional, with varying degrees of intensity—ranging from **profound sadness** to **mild contentment**. **Fuzzy set theory** serves as a valuable framework for representing the emotions and actions of humanoid robots. The modeling of **facial features**, including the **mouth, eyes, and other expressive components** of humanoid robots, requires **fuzzy logic** rather than rigid classical logic models [1, 12]. If humanoid robots are to behave in a way that closely resembles **human behavior**, the use of **fuzzy set extensions** in modeling is **essential**, as they provide a **more flexible and realistic representation** of human emotions and decision-making processes. In **classical fuzzy logic** [13], each element has **only one membership degree**, with **the non-membership degree being its complement**. However, **fuzzy set extensions** offer **greater expressiveness** in modeling uncertainty and ambiguity, which are inherent in human emotions. A **Scopus** search for *"Fuzzy Humanoid Robot"* yields **484 results**, highlighting the growing academic interest in this field. Figures 1 and 3 illustrate the key findings of the **literature review on humanoid robots**.

The distribution of papers by year is shown in Fig. 1. The first three articles were published in 1997. The year 2013 had the highest number of publications, accounting for 7.8% of the total. The distribution of publications on humanoid robots by

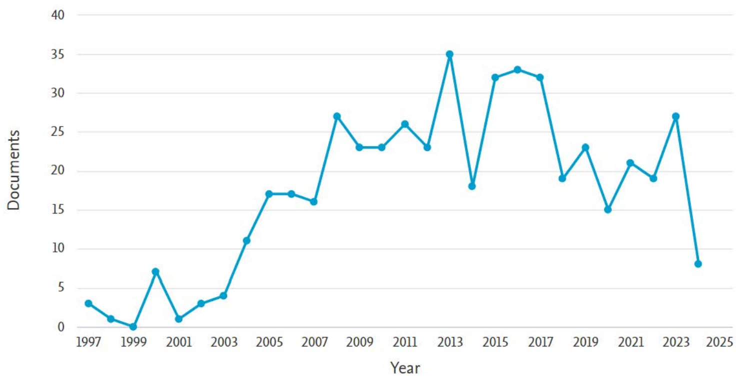

Fig. 1. Distribution of humanoid robot papers with respect to years.

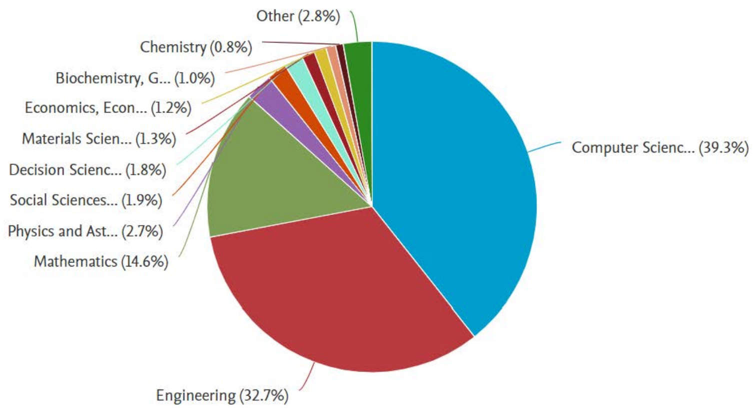

Fig. 2. Distribution of humanoid robot papers with respect to subject areas.

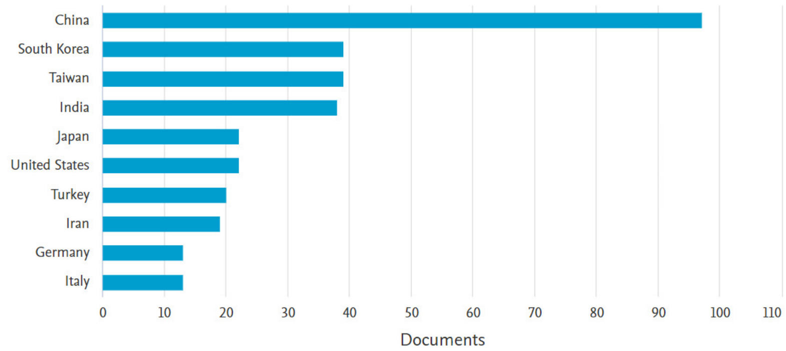

Fig. 3. Distribution of humanoid robot papers by their countries.

topic domain is illustrated in Fig. 2. The fields with the most publications include **engineering, computer science**, and **mathematics**. The distribution of publications on humanoid robots according to their country of origin is depicted in Fig. 3. China is at the forefront of humanoid robot research, followed by South Korea and Taiwan. Given that human behavior is the result of a complex cognitive system, defining uncertainty requires multiple factors. This is precisely what extensions of classical fuzzy logic aim to achieve. Since these extensions are better suited for modeling human behavior than conventional fuzzy logic [13], we introduce them here. Several extensions of fuzzy set theory have been proposed, including:

- **Intuitionistic fuzzy sets (IFSs)** (Atanassov, 1986 [14]),
- **Interval-valued IFSs** (Atanassov & Gargov, 1986 [15]),

- **Neutrosophic fuzzy sets** (Smarandache, 1998 [16]),
- **Hesitant fuzzy sets** (Torra, 2010 [17]),
- **Pythagorean fuzzy sets** (Yager, 2013 [18]),
- **Picture fuzzy sets** (Cuong, 2013 [19]),
- **q-rung orthopair fuzzy sets** (Yager, 2017 [20]),
- **Cubic sets** (Mahmood, Abdullah & Bilal, 2017 [21]),
- **Spherical fuzzy sets** (Kutlu Gündogdu & Kahraman, 2019 [22]),
- **Fermatean fuzzy sets** (Senapati & Yager, 2020 [23]).

The study [24] provides a detailed comparison of the extensions of IFSs. The authors of [24] demonstrated that an IFS can fully characterize a hesitant fuzzy set [17]. Additionally, they showed that **Picture fuzzy sets** [19], **Cubic sets** [21], **Neutrosophic fuzzy sets** [16], and **Support-intuitionistic fuzzy sets** [25] can all be represented by interval-valued IFSs (IVIFSs) [15]. In recent years, two further generalizations of interval-valued intuitionistic fuzzy sets have been proposed: **circular** (C-IFSs, [26]) and **elliptic** (E-IFSs, [11]). A C-IFS [26] is a circle of radius r that centers the membership and non-membership degrees of IFSs [14]. An E-IFS [11] represents the membership and non-membership degrees of IFSs [14] within an ellipse defined by its semi-major and semi-minor axes.

There are relatively few studies on fuzzy or intelligent humanoid robot modeling. In 2003, Kats and Vukobratovic [27] reviewed intelligent control methods for humanoid robots. Wong et al. (2008) [28] investigated humanoid robot fuzzy control. Fang et al. (2019) [29] explored emotional learning models in fuzzy neural networks. Kahraman and Bolturk (2021) [1] presented fuzzy set-based models for simulating a humanoid robot's emotions and movements. The facial expressions of a humanoid robot were modeled in [30] using Pythagorean fuzzy sets [18] and IFSs [14], based on emotional intensity. A humanoid robot's facial expressions must be contextually appropriate. Humans possess over **17 distinct facial expressions**, which cannot be fully captured using discrete sets [31]. These expressions include happily surprised, happily disgusted, sadly fearful, sadly angry, sadly surprised, sadly disqusted, fearfully angry, fearfully surprised, fearfully disgusted, angrily surprised, angrily disgusted, disgustedly surprised, appalled, hateful, and awed. There are many facial muscle articulations (also known as action units, or AUs) connected to every emotional category. A distinctive characteristic of the visible image is produced by a group of articulations of the facial muscles known as AU. Each emotional category corresponds to a set of facial muscle articulations, known as **action units (AUs)**. AUs define the characteristic features of facial expressions and contribute to the visible representation of emotions.

The classical functions $0.2x^2 - 3\ \{-2 < x < 2\}$ and $0.5x^2 - 3\ \{-2 < x < 2\}$ can be utilized. As shown in Figs. 4 and Fig. 5 [32], mouth mimics of the lower lip from medium strong smile and strong smile are attempted to model. It is possible to

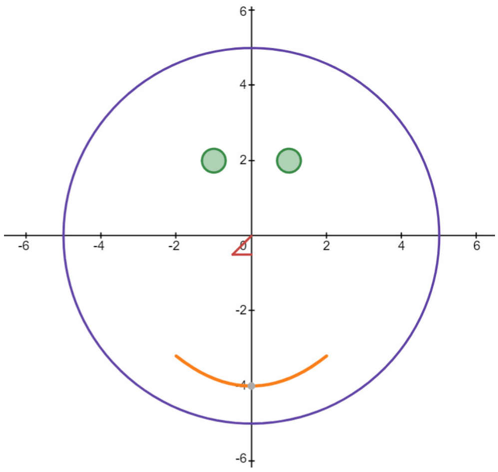

Fig. 4. Smile using the mathematical function $0.2x^2 - 3\ \{-2 < x < 2\}$.

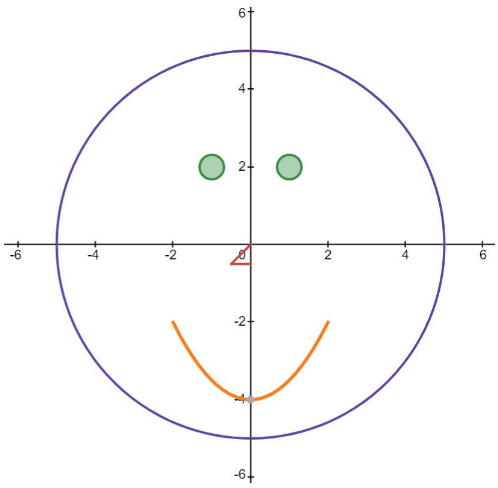

Fig. 5. Smile using the mathematical function $0.5x^2 - 3\ \{-2 < x < 2\}$

present the crisp function $f_{\text{lower lip}}$ using the following equation: (1) [33]

(1) $$f_{\text{lower lip}} = c_1 x^2 - 3, -2 \le x \le 2\,.$$

The humanoid robot should smile between a very weak and a medium strong smile during an event when the parameter c_1 varies between 0.1 and 0.36. The humanoid robot should grin between a medium-strong and a very-strong smile at an event when the parameter c_1 changes between 0.5 and 0.9 [32].

3 Essential Terms for the E-IFQs and E-IFIMs

The elliptic IFSs are one of the newest developments of IFSs, having been introduced by Atanasov in 2021 [11].

3.1 Elliptic Intuitionistic Fuzzy Quads (E-IFQs)

According to [35], an elliptic intuitionistic fuzzy quad (E-IFQ) is an object of the following form: $\langle a(p), b(p); u, v\rangle = \langle \mu(p), \nu(p); u, v\rangle$, where $a(p) + b(p) = \mu(p) + \nu(p) \le 1$. The assertion p has two degrees of truth and falsehood, which are $a(p)(\mu(p))$ and $b(p)(\nu(p))$, and $a(p) + b(p) \le 1$. $u, v \in [0, \sqrt{2}]$ are the semi-major and semi-minor axes of the ellipse. Let us consider the following two E-IFQs be given: $x_{u_1,v_1} = \langle a, b; u_1, v_1\rangle$ and $y_{u_2,v_2} = \langle c, d; u_2, v_2\rangle$. We are going to define an operation called $* \in \{\min, \max\}$.

$$\begin{aligned}
x \vee_{1*} y &= \langle \max(a, c), \min(b, d); *(u_1, u_2), *(v_1, v_2)\rangle;\\
x \wedge_{1*} y &= \langle \min(a, c), \max(b, d); *(u_1, u_2), *(v_1, v_2)\rangle;\\
x \wedge_{2*} y &= x + y = \langle a + c - a.c, b.d; *(u_1, u_2), *(v_1, v_2)\rangle;\\
x \vee_{2*} y &= x.y = \langle a.c, b + d - b.d; *(u_1, u_2), *(v_1, v_2)\rangle;\\
\alpha.x &= \langle 1 - (1 - a)^\alpha, b^\alpha; *(u_1, u_2), *(v_1, v_2)\rangle (\text{for } \alpha > 0);\\
x -_* y &= \langle \max(0, a - c), \min(1, b + d, 1 - a + c); *(u_1, u_2), *(v_1, v_2)\rangle.
\end{aligned}$$

The following relation for E-IFP comparison is proposed in [11].

(2) $$\begin{aligned}
x &\ge y && \text{if } a \ge c, b \le d, u_1 \le v_1 \text{ and } u_2 \le v_2;\\
x &\ge_\Box y && \text{if } a \ge c;\\
x &\ge_\Diamond y && \text{if } b \le d;
\end{aligned}$$

Two IF options, $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$, can be ranked in the following manner: If $R_x^{\text{elliptic}} \le R_y^{\text{elliptic}}$, then x is a superior option [35], when the separation between the perfect, beneficial option is $R_x^{\text{elliptic}} = \frac{1}{6}(2 - a - b)\big(|\sqrt{2} - u_1| + |\sqrt{2} - v_1| + |1 - a|\big)$ for $\langle 1, 0; \sqrt{2}, \sqrt{2}\rangle$ to x.

There is a fixed universe E for us. Assume we have an E-IFP $x = \langle a, b; u_1, v_1 \rangle$. Let us expand [36]'s methods for de-intuitionistic fuzzification of circular IF triples to include the following steps:

- The first step involves placing an elliptic form in the center of the Intuitionistic Fuzzy Interpretation Triangle next to each member $x \in E$. The center can be transformed in fuzzy pair by using Eq. 3 [36]:

$$\left\langle \frac{a}{a+b}, \frac{b}{b+a} \right\rangle \tag{3}$$

- The second procedure involves using an elliptic form to juxtaposition three points, $X(x)$, $X_L(x)$, and $X_R(x)$, across the hypotenuse of the Intuitionistic Fuzzy Interpretation Triangle, with coordinates for each element $x \in E$.

$$X(x) = \left\langle \frac{1+a-b}{2}, \frac{1-a+b}{2} \right\rangle,$$

$$X(L) = \left\langle \max\left(0, \frac{1+a-b-\sqrt{2}r_1 - \sqrt{2}r_2}{2}\right), \min\left(1, \frac{1-a+b+\sqrt{2}r_1 + \sqrt{2}r_2}{2}\right) \right\rangle$$

and

$$X(R) = \left\langle \min\left(0, \frac{1+a-b+\sqrt{2}r_1 + \sqrt{2}r_2}{2}\right), \max\left(1, \frac{1-a+b-\sqrt{2}r_1 - \sqrt{2}r_2}{2}\right) \right\rangle.$$

3.2 Description, usage, and relations to elliptic IF IMs (E-IFIMs)

Index matrices (IMs) were first introduced in 1987 as a method for describing transitions in generalized networks (see [37] and [38]). The constant set of indices, denoted by $\mathcal{I}$, will be used in the following discussion. A two-dimensional E-IFIM can be described using the following definition:

$$A = [K, L, \langle \mu_{k_i,l_j}, \nu_{k_i,l_j}; rf_{k_i,l_j}, rs_{k_i,l_j} \rangle]$$

$$\begin{array}{c|ccccc} & l_1 & \ldots & l_j & \ldots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1}; rf_{k_1,l_1,h_g}, rs_{k_1,l_1,h_g} \rangle & \ldots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j}; rf_{k_1,l_j}, rs_{k_1,l_j} \rangle & \ldots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n}; rf_{k_1,l_n}, rs_{k_1,l_n} \rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1}; rf_{k_m,l_1}, rs_{k_m,l_1} \rangle & \ldots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j}; rf_{k_m,l_j}, rs_{k_m,l_j} \rangle & \ldots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n}; rf_{k_m,l_n}, rs_{k_m,l_n} \rangle \end{array},$$

where the index sets K and L are subsets of the set $\mathcal{I}$ [35].

Similar to the definition given in [39], the 3-D E-IFIM definition builds upon the 2-D E-IFIM definition. E-IFIMs $A = [K, L, \langle \mu_{k_i,l_j}, \nu_{k_i,l_j}; rf_{k_i,l_j}, rs_{k_i,l_j} \rangle]$ and $B = [P, Q, \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s}; \delta f_{p_r,q_s}, \delta s_{p_r,q_s}]$ can be used with a variety of methods, relationships, and operators, defined in [35].

4 Humanoid robot mouth expression modeling using elliptic intuitionistic fuzzy sets

We propose the following E-IF algorithm to identify various humanoid robot lower lip expressions by extending the IF method from [33] and incorporating the expert-based approach from [35].

Step 1. To determine different parameter values c_j $(j = 1, \ldots, n)$ for the function representing humanoid robot lower lip expressions $\{k_1, \ldots, k_i, \ldots, k_m\}$, a professional evaluation by experts $\{d_1, \ldots, d_s, \ldots, d_D\}$ is required.

An intuitionistic fuzzy (IF) index matrix $EVR[K, C, E, \{ev_{k_i,c_j,d_s}\}]$, is then constructed: where $K = \{k_1, \ldots, k_i, \ldots, k_m\}$ represents the set of different lower lip expressions, $C = \{c_1, \ldots, c_j, \ldots, c_n\}$ denotes the set of expression parameters and $E = \{d_1, \ldots, d_s, \ldots, d_D\}$ corresponds to the set of expert evaluators. The robot's bottom lip is represented in its various states within the set K. One parameter of the robot's lower lip function is denoted as c_j $(j = 1, \ldots, n)$. The competency coefficients of the experts, denoted as r_s $(s \in E)$, need to be specified. The index matrix (IM) is constructed as follows:

$$EVR^*[K, C, E, \{ev^*{}_{k_i,c_j,d_s}\}]$$
$$= r_1 \cdot pr_{K,C,d_1} EV \oplus_{(\circ_1,\circ_2)} r_2 \cdot pr_{K,C,d_2} EV \oplus_{(\circ_1,\circ_2)} \cdots \oplus_{(\circ_1,\circ_2)} r_D \cdot pr_{K,C,d_D} EV.$$

The final expert evaluation result is given by

$$EVR := EVR^*, \text{ where } (evr_{k_i,c_j,d_s} = evr^*_{k_i,c_j,d_s}, \ \forall k_i \in K, \forall c_j \in C, \forall d_s \in E).$$

Since certain uncontrolled factors may have changed, the experts may be uncertain about their conclusions. To address this uncertainty, the following procedures are used to convert the assessments into intuitionistic fuzzy parameters (IFPs). Let us define the current set of intervals for expert ratings of each applicant based on all criteria at a given moment h_f as: $[p^{1,f}_{k_i,c_j,d_s}; p^{2,f}_{k_i,c_j,d_s}]$ $\forall k_i \in K, \forall c_j \in C, \forall d_s \in E$.

$$A_{\min,i,j,s,f} = \min_{1 \le i \le m, 1 \le j \le n, 1 \le s \le D} p^{1,f}_{k_i,c_j,d_s} < \max_{1 \le i \le m, 1 \le j \le n, 1 \le s \le D} p^{2,f}_{k_i,c_j,d_s} = A_{\max,i,j,s,f}\,. \tag{4}$$

Between $[p^{1,f}_{k_i,c_j,d_s}; p^{2,f}_{k_i,c_j,d_s}]$, at a time point h_f, we build the c_j-th parameter and the d_s-th expert evaluation for the k_i-th lower lip event using the format of IFP [34] as follows:

$$\begin{aligned} \mu_{k_i,c_j,d_s,h_f} &= \frac{p^{1,f}_{k_i,c_j,d_s} - A_{\min,i,j,s,f}}{A_{\max,i,j,s,f} - A_{\min,i,j,s,f}}\,, \\ \nu_{k_i,c_j,d_s,h_f} &= \frac{A_{\max,i,j,s,f} - p^{2,f}_{k_i,c_j,d_s}}{A_{\max,i,j,s,f} - A_{\min,i,j,s,f}}\,. \end{aligned} \tag{5}$$

If the expert assessments that meet the conditions are IFPs, this change is not accomplished. Some of the experts' conclusions can be incorrect from an IF standpoint. A variety of techniques for altering the evaluations of flawed experts are examined in [34]. Then, an IF index matrix $EV[K, C, E, \{ev_{k_i,c_j,d_s}\}], K = \{k_1, \dots, k_i. \dots, k_m\}$, $C = \{c_1, \dots, c_j, \dots, c_n\}$ and $E = \{d_1, \dots, d_s, \dots, d_D\}$ is constructed by transforming the IM EVR using (5). The elements $\{ev_{k_i,c_j,d_s}\} = \langle \mu_{k_i,c_j,d_s}, \nu_{k_i,c_j,d_s} \rangle$ (for $1 \le i \le m, 1 \le j \le n, 1 \le s \le D$) of the matrix EV depend on the uncertainty and are IF valuations of the d_s-th expert for the k_i-th lower lip expression by the c_j-th parameter of the lower lip function of the robot. $EV := EV^*(ev_{k_i,c_j,d_s} = ev^*_{k_i,c_j,d_s}, \forall k_i \in K, \forall c_j \in C, \forall d_s \in E)$. Next, we go on to *Step 2*.

Step 2. The degrees of membership and non-membership of the E-IFQs are determined by the elements of the matrix EV using the three aggregation operations $\alpha_{K,\#_1,*}$, $\alpha_{K,\#_2,*}$, and $\alpha_{K,\#_3,*}$. These operations provide evaluations of the k_i-th lower lip expression based on the c_j-th parameter $(j = 1, \dots, j, \dots, n)$ of the lower lip function at a given moment $h_f \notin E$:

$$PI_{\min}[K, h_f, C, \{pi_{\min k_i,h_f,c_g}\}] = \alpha_{E,\#_1}(EV^*, h_f)$$

$$= \left\{ \begin{array}{c|c} c_j & h_f \\ \hline k_1 & \underset{s=1}{\overset{D}{\#_1}} \langle \mu_{k_1,c_j,d_s}, \nu_{k_1,c_j,d_s} \rangle \\ \vdots & \vdots \\ k_m & \underset{s=1}{\overset{D}{\#_1}} \langle \mu_{k_m,c_j,d_s}, \nu_{k_m,c_j,d_s} \rangle \end{array} \middle| \; c_j \in C \right\};$$

$$PI_{\max}[K, h_f, C, \{pi_{\max k_i,h_f,c_g}\}] = \alpha_{E,\#_3}(EV^*, h_f)$$

$$= \left\{ \begin{array}{c|c} c_j & h_f \\ \hline k_1 & \underset{s=1}{\overset{D}{\#_3}} \langle \mu_{k_1,c_j,d_s}, \nu_{k_1,c_j,d_s} \rangle \\ \vdots & \vdots \\ k_m & \underset{s=3}{\overset{D}{\#_1}} \langle \mu_{k_m,c_j,d_s}, \nu_{k_m,c_j,d_s} \rangle \end{array} \middle| \; c_j \in C \right\}$$

$$PI* = PI_{\min} \oplus_{(\circ_1,\circ_2,*)} PI_{\max}.$$

Next, as elements in a matrix, the centers of the E-IFQs utilized to assess the parameters for the lower lip function are expressed as follows:

$$PI[K, h_f, C, \{pi_{k_i,h_f,c_g}\}] = \alpha_{E,\#_2}(PI^*, h_f), (h_f \notin E).$$

E-IFIM $A[K, C, h_f\{a_{k_i,c_j,h_f}\}]$ can now be calculated, which reflects the most recent evaluations of the settings for the robot mouth in various scenarios utilizing the methods from [35]:

$$\begin{array}{c|ccc} h_f & c_1 & \dots & c_n \\ \hline k_1 & \langle \mu^a_{k_1,c_1}, \nu^a_{k_1,c_1}; rf^a_{k_1,c_1}, rs^a_{k_1,c_1} \rangle & \dots & \langle \mu^a_{k_1,c_n}, \nu^a_{k_1,c_n}; rf^a_{k_1,c_n}, rs^a_{k_1,c_n} \rangle \\ \vdots & \vdots & \dots & \vdots \\ k_m & \langle \mu^a_{k_m,c_1}, \nu^a_{k_m,c_1}; rf^a_{k_m,c_1}, rs^a_{k_m,c_1} \rangle & \dots & \langle \mu^a_{k_m,c_n}, \nu^a_{k_m,c_n}; rf^a_{k_m,c_n}, rs^a_{k_m,c_n} \rangle \end{array},$$

where $K = \{k_1, \dots, k_i, \dots, k_m\}, i = 1, \dots, m$; $C = \{c_1, \dots, c_j, \dots, c_n\}, j = 1, \dots, n$; its elements a_{k_i,c_j,h_f} (for $i = 1, \dots, m; j = 1, \dots, n$) are created as E-IFQs by transforming the IFPs ${pi^T}_{k_i,c_j,h_f}$ using the following steps:

for $j = 1$ to n, $i = 1$ to m

$$\begin{aligned} \mu_{k_i,c_j,h_f} &= \mu^{pi^{\text{ave}}}_{k_i,c_j,h_f}; \nu_{k_i,c_j,h_f} = \nu^{pi^{\text{ave}}}_{k_i,c_j,h_f}, \\ rf_{k_i,c_j,h_f} &= \sqrt{PI^{\text{leftCB},\mu^2}_{k_i,c_j,h_f} + \left\{ \frac{PI^{\text{rightCB},\mu^2}_{k_i,c_j,h_f} - PI^{\text{leftCB},\mu^2}_{k_i,c_j,h_f}}{PI^{\text{rightCB},\nu^2}_{k_i,c_j,h_f} - PI^{\text{leftCB},\nu^2}_{k_i,c_j,h_f}} \right\}^2 . PI^{\text{leftCB},\nu^2}_{k_i,c_j,h_f}} \\ rs^a_{k_i,c_j,h_f} &= \sqrt{PI^{\text{leftCB},\mu^2}_{k_i,c_j,h_f} . \left\{ \frac{PI^{\text{rightCB},\nu^2}_{k_i,c_j,h_f} - PI^{\text{leftCB},\nu^2}_{k_i,c_j,h_f}}{PI^{\text{rightCB},\mu^2}_{k_i,c_j,h_f} - PI^{\text{leftCB},\mu^2}_{k_i,c_j,h_f}} \right\}^2 + PI^{\text{leftCB},\nu^2}_{k_i,c_j,h_f}}. \end{aligned}$$

Next, we go on to *Step 3*.

Step 3. Now, a 3-D E-IFIM PK is created, and the experts decide how to weight each evaluation parameter for the lower lip function in an instant h_f:

$$PK[K, C, h_f, \{pk_{k_i,c_j,h_f}\}] = \begin{array}{c|ccc} h_f & c_1 & \dots & c_n \\ \hline k_1 & pk_{k_1,c_1,h_f} & \dots & pk_{k_1,c_n,h_f} \\ \vdots & \vdots & \dots & \vdots \\ k_m & pk_{k_m,c_1,h_f} & \dots & pk_{k_m,c_n,h_f} \end{array},$$

where $K = \{k_1, \dots, k_i, \dots, k_m\}, C = \{c_1, \dots, c_j, \dots, c_n\}$ and the elements pk_{k_i,c_j,h_f} are E-IFQs. The evaluation E-IFIM is given by

(6) $FI[K, C, h_f, \{fi_{k_i,c_j,h_f}\}] = A \otimes_{(\circ_1,\circ_2,*,*)} PK, \quad \forall i = 1, \dots, m; \quad \forall j = 1, \dots, n.$

This matrix contains all E-IF estimates for the k_i-th lower lip event corresponding to the c_j-th parameter of the lower lip function. The choice of the aggregation operator $*$ depends on the level of uncertainty: if there is more uncertainty, then $* = \max$; if there is less uncertainty, then $* = \min$. The second technique can be applied to the de-intuitionistic fuzzification of: $fi_{k_i,l_j,h_f} = \langle \mu^y_{k_i,l_j,h_f}, \nu^y_{k_i,l_j,h_f}; rf^a_{k_i,c_j,h_f}, rs^a_{k_i,c_j,h_f} \rangle$ for $(i = 1, \ldots, m; j = 1, \ldots, n)$ from the previous section. Let us obtain

$$\langle \mu^{y,\text{fuzzy}}_{k_i,l_j,h_f}, \nu^{y,\text{fuzzy}}_{k_i,l_j,h_f} \rangle, \quad \langle \mu^{y,\text{fuzzy}_{\text{Left}}}_{k_i,l_j,h_f}, \nu^{y,\text{fuzzy}_{\text{Left}}}_{k_i,l_j,h_f} \rangle \quad \text{and} \quad \langle \mu^{y,\text{fuzzy}_{\text{Right}}}_{k_i,l_j,h_f}, \nu^{y,\text{fuzzy}_{\text{Right}}}_{k_i,l_j,h_f} \rangle .$$

Following that, we defuzzify the fuzzy pairings into distinct numerical values: $p^{fi}_{k_i,c_j,h_f}$, $p^{fi_{\text{left}}}_{k_i,c_j,h_f}$ and $p^{fi_{\text{right}}}_{k_i,c_j,h_f}$ for $(i = 1, \ldots, m; j = 1, \ldots, n)$ using Eq. 5.

5 Case Study: Using Elliptic Intuitionistic Fuzzy Sets to Express Mouth Movements in Humanoid Robots

Let us assume that the robot's lower lip can be in three states: k_1 - between a strong smile and a super strong smile; k_2 - between a medium strong smile and a strong smile and k_3 - between a very weak smile and a medium strong smile. Assume that the evaluation process for the parameter c_1 in the E-IF function (1) for the lower lip involves three experts. The corresponding assessment is given as follows:

$$\left\{ \begin{array}{c|c} d_1 & c_1 \\ \hline k_1 & \langle 0.80, 0.20 \rangle \\ k_2 & \langle 0.50, 0.20 \rangle \\ k_3 & \langle 0.10, 0.20 \rangle \end{array}, \begin{array}{c|c} d_2 & c_1 \\ \hline k_1 & \langle 0.90, 0.10 \rangle \\ k_2 & \langle 0.60, 0.10 \rangle \\ k_3 & \langle 0.20, 0.10 \rangle \end{array}, \begin{array}{c|c} d_3 & c_1 \\ \hline k_1 & \langle 0.70, 0.30 \rangle \\ k_2 & \langle 0.40, 0.30 \rangle \\ k_3 & \langle 0.30, 0.30 \rangle \end{array} \right\}$$

Then, these are the rank coefficients of the experts:

$$\{r_1, r_2, r_3\} = \{\langle 0.80, 0.10 \rangle, \langle 0.70, 0.10 \rangle, \langle 0.90, 0.10 \rangle\}.$$

The evaluation IM $EV^*[K, C, E, \{ev^*\}]$ is made using the subsequent procedures:

$$EV^* = r_1 pr_{K,C,d_1} EV \oplus_{(\circ_1,\circ_2)} r_2 pr_{K,C,d_2} EV \oplus_{(\circ_1,\circ_2)} r_3 pr_{K,C,d_3} EV; EV := EV^*$$

Then, the IMs are created: $PI* = PI_{\min} \oplus_{(\circ_1,\circ_2,*)} PI_{\max}$ and $PI[K, h_f, C, \{pi_{k_i,h_f,c_g}\}] = \alpha_{E,\#_2}(PI^*, h_f), (h_f \notin E)$ whose components are the center coordinates of the E-IFQs as determined by various lower lip events for the parameter c_1. Let's compute E-IFIM $A[K, C, h_f\{a_{k_i,c_g,h_f}\}]$ now. It shows the latest evaluations of the parameter c_1 based on various lower lip events.

(7)

h_f	c_1
k_1	$\langle 0.8, 0.2; 0.1, 0.05\rangle$
k_2	$\langle 0.5, 0.2; 0.1, 0.05\rangle$
k_3	$\langle 0.2, 0.2; 0.1, 0.05\rangle$

At this point, a 3-D E-IFIM PK determines the priority of the parameter c_1 at a moment h_f for the k_i-th lower lip event:

(8) $PK[K, C, h_f, \{pk_{k_i,c_1,h_f}\}] =$

h_f	c_1
k_1	$\langle 0.90, 0.10; 0.02, 0.01\rangle$
k_2	$\langle 0.90, 0.10; 0.02, 0.01\rangle$
k_3	$\langle 0.90, 0.10; 0.02, 0.01\rangle$

The evaluation E-IFIM is given by: $FI[K, C, h_f, \{fi_{k_i,c_1,h_f}\}] = A \otimes_{(\circ_1,\circ_2,\min,\min)} PK$, for $1 \leq i \leq 3$. This matrix includes the estimates of the k_i-th lower lip state based on the optimistic case. The evaluation E-IFIM is given by:

(9) $$FI[K, C, h_f, \{fi_{k_i,c_1,h_f}\}] = A \otimes_{(\circ_1,\circ_2,\min,\min)} PK, \quad \text{for } 1 \leq i \leq 3.$$

The resulting evaluation matrix FI is:

(10) $FI =$

h_f	c_1
k_1	$\langle 0.72, 0.28; 0.02, 0.01\rangle$
k_2	$\langle 0.45, 0.28; 0.02, 0.01\rangle$
k_3	$\langle 0.18, 0.28; 0.02, 0.01\rangle$

Let x be a member of the set $[-2, 2]$. In the interval of fuzzy pairs, the E-IFQ

$$\langle 0.72, 0.28; 0.02, 0.01\rangle$$

is changed from $\langle 0.7, 0.3\rangle$ to $\langle 0.74, 0.26\rangle$. This interval is then defuzzified as $[0.8, 0.96]$. Thus, the function for the first lower lip state is obtained as: $f_{k_1} = [0.8, 0.96]x^2 - 3, \quad -2 \leq x \leq 2$. The procedures for the remaining two lower lip states are analogous, yielding: $f_{k_2} = [0.26, 0.42]x^2 - 3, \quad f_{k_3} = [-0.12, -0.28]x^2 - 3$.

6 Discussion

The results demonstrate the effectiveness of E-IFSs in modeling humanoid robot mouth expressions that accurately reflect different emotional states. By leveraging the unique properties of E-IFSs, our approach captures emotional nuances more flexibly and realistically than traditional methods. A key advantage of the E-IFS model is its ability to handle uncertainty and imprecision in human emotions. Unlike fixed geometric transformations or predefined templates, E-IFS enables dynamic adjustments of mouth shapes based on varying emotional intensities. This adaptability is crucial for humanoid robots interacting in diverse real-world environments. The integration of elliptic parameters allows smooth transitions between emotions, while E-IF logic provides a richer and more nuanced representation by incorporating membership, non-membership, and hesitation degrees. Despite its benefits, the E-IFS model has some limitations. Its computational complexity, $O(Dm^2n^2)$, may pose challenges for real-time applications, requiring further optimization for resource-constrained environments. Future research should explore the model's adaptability across different humanoid designs and its application in multi-modal emotion representation, including upper lip, eye, and eyebrow expressions. Another important direction is evaluating user perception and acceptance. While the technical performance of the E-IFS model is promising, user studies assessing the naturalness and emotional resonance of the robot's expressions will be essential for refining and validating the model.

7 Conclusion

This paper presented a novel approach for modeling humanoid robot mouth expressions using E-IFS, providing a flexible framework for representing the complexity of human emotions. By integrating E-IF logic, the model effectively generates realistic and contextually appropriate mouth expressions, enhancing human-robot interaction. The E-IFS-based model offers key advantages over traditional methods, particularly in handling uncertainty and adapting to varying emotional intensities. Its ability to capture nuanced expressions makes it a promising step toward more natural and intuitive robotic communication. In conclusion, developing emotionally expressive humanoid robots is essential for improving human-robot interactions. The E-IFS approach contributes to this goal by creating robots that are both functionally effective and emotionally engaging. Future work will address current limitations and explore its broader applications in multi-modal emotion representation and user perception.

Acknowledgements

Work on Sections 1–4 is supported by the Bourgas State University "Prof. Dr. Assen Zlatarov" through project Ref. No. NIX-486/2023: "*Application of Intuitionistic Fuzzy Logic in Decision Making*".

Work on Sections 5–7 is supported by the same university through project Ref. No. NIX-482/2023: "*Modeling Management Decisions with New Analysis Tools in a Fuzzy Environment*".

REFERENCES

[1] C. KAHRAMAN, E. BOLTURK (2021) "Toward Humanoid Robots: The Role of Fuzzy Sets". Studies in Systems, Decision and Control, Springer, Cham **344**

[2] G. TZVETKOVA (2014) Robonaut 2: Mission, Technologies, Perspectives. *Journal of Theoretical and Applied Mechanics, Sofia* **44** (1) 97-102

[3] T. SHARARI (2015) Building and Programming a Smart Robotic System for Distinguishing Objects Based on Their Shape and Colour. *Journal of Theoretical and Applied Mechanics, Sofia* **45** (1) 53-68

[4] T. TAWARA, Y. OKUMURA, M. SHIMIZU, K. TOMIYAMA (2001) Design and construction of a series of compact humanoid robots and development of biped walk control strategies. *Robot. Auton. Syst.* **37** (2-3) 81-100

[5] L. HAWLEY, W. SULEIMAN (2019) Control framework for cooperative object transportation by two humanoid robots. *Robot. Auton. Syst.* **115** 1-16

[6] J. CARPENTER, J. M. DAVIS, N. ERWIN-STEWART, N., T. R. LEE, J. D. BRANSFORD, N. VYE (2009) Gender representation and humanoid robots designed for domestic use. *Int. J. Soc. Robot.* **1** (3) 261

[7] C. BREAZEAL (2003) Emotion and sociable humanoid robots. *Int. J. Hum. Comput. Stud.* **59** (1-2) 119-155

[8] B. ROBINS, K. DAUTENHAHN, R. TE BOEKHORST, A. BILLARD, A. (2005) Robotic assistants in therapy and education of children with autism: can a small humanoid robot help encourage social interaction skills? *Univ. Access Inf. Soc.* **4** (2) 105-120

[9] D. BELANCHE, L. V. CASALÓ, C. FLAVIÁN (2020) Customers acceptance of humanoid robots in services: the moderating role of risk aversion *Marketing and Smart Technologies* 449-458-120

[10] J. DING, J. Y. L. SHI (2017) Kinect microphone array-based speech and speaker recognition for the exhibition control of humanoid robots. *Comput. Electr. Eng.* **62** 719-729

[11] ATANASSOV, K. (2021) Elliptic Intuitionistic Fuzzy Sets. *Comptes rendus de l'Académie bulgare des Sciences.* **74** (65) 812-819

[12] R. ZALL,M. R. KANGAVARI (2022) Comparative Analytical Survey on Cognitive Agents with Emotional Intelligence. *Cogn. Comput.* **14** 1223-1246

[13] L. ZADEH (1965) Fuzzy Sets. *Information and Control* **8** (3) 338-353

[14] K. ATANASSOV (1983) Intuitionistic fuzzy sets. In: *Proceedings of VII ITKR's Session*, Sofia, 1983.

[15] K. ATANASSOV, G. GARGOV (1989) Interval-valued intuitionistic fuzzy sets. *Fuzzy sets and systems* **31** (3) 343-349

[16] F. SMARANDACHE, (1998) "Neutrosophy. Neutrosophic Probability". Set, and Logic, Amer. Res. Press, Rehoboth, USA pp. 207–223.

[17] TORRA V. (2010) Hesitant fuzzy sets. *Intern. Journal of Intelligent Systems* **25** (6) 529-539

[18] R. R. YAGER (2013) Pythagorean fuzzy subsets. *Joint IFSA World Congress and NAFIPS Annual Meeting* **2013** 57-61

[19] C. B. Cuong, V. Kreinovich (2013) Picture fuzzy sets - a new concept for computational intelligence problems. In: *Proceedings of the Third World Congress on Information and Communication Technologies WICT'2013*, Hanoi, Vietnam, pp. 1-6.

[20] R. R. Yager (2017) Generalized Orthopair Fuzzy Sets. *IEEE Transactions on Fuzzy Systems* **25** (5) 1222-1230

[21] T. Mahmood, S. -ur-Rashid Abdullah, M. Bilal (2017) Multicriteria decision making based on a cubic set. *Journal of New Theory* **2017** (16) 1-9

[22] F. Kutlu Gündoğdu, C. Kahraman (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of Intelligent Fuzzy Systems* **36** (1) 33-352

[23] T. Senapati, R. R. Yager (2020) Fermatean fuzzy sets. *Journal of Ambient Intelligence and Humanized Computing* **11** 663-674

[24] P. Vassilev, K. Atanassov (2019) "Modifications and extensions of Intuitionistic Fuzzy Sets". Prof. Marin Drinov, Academic Publishing House, Sofia

[25] X. T. Nguyen, V. D. Nguyen (2015) Support-Intuitionistic Fuzzy Set: A New Concept for Soft Computing. *I.J. Intelligent Systems and Applications* **2015** (04) 11-16

[26] K. Atanassov (2020) Circular Intuitionistic Fuzzy Sets. *Journal of Intelligent & Fuzzy Systems* **39** (5) 5981-5986

[27] D. Katic, M. Vukobratovic (2003) Survey of intelligent control techniques for humanoid robots. *J. Intell. Rob. Syst.* **37** (2) 117-141

[28] C.C. Wong, C. T. Cheng, K. H. Huang, Y. T. Yang (2008) Fuzzy control of humanoid robot for obstacle avoidance. *Int. J. Fuzzy Syst.* **10** (1) 117-141

[29] W. Fang, F. Chao, C. M. Lin, L. Yang, C. Shang, C. Zhou (2019) An improved fuzzy brain emotional learning model network controller for humanoid robots. *Front. Neurorobotics* **13** (2) 1-16

[30] C. Kahraman, E. Bolturk, S. C. Onar, B. Oztaysi (2021) Modeling Humanoid Robots Using Fuzzy Set Extensions. In: C. Kahraman, E. Bolturk (eds) "Toward Humanoid Robots: The Role of Fuzzy Sets. Studies in Systems, Decision and Control". Studies in Systems, Decision and Control,Springer, Cham, vol. **344**, pp. 99-119

[31] Sh. Du, A. Martinez (2015) Compound facial expressions of emotion: From basic research to clinical applications. *Dialogues in Clinical Neuroscience* **17** (4) 443-445

[32] Desmos calculator (accessed Oct. 16, 2024) https://www.desmos.com/calculator/8gbvtwldqr.

[33] C. Kahraman, S. C. Onar, B. Oztaysi, I. Otay (2021) Modeling Humanoid Robots Mimics Using Intuitionistic Fuzzy Sets. In: C. Kahraman, S. Cevic Onar, B. Oztaysi, I. Otay (eds) "Intelligent and Fuzzy Techniques: Smart and Innovative Solutions. INFUS 2020. ". Advances in Intelligent Systems and Computing, Springer, Cham, vol. **1197**, pp. 339-346

[34] K. Atanassov (2012) "On Intuitionistic Fuzzy Sets Theory". STUDFUZZ, Springer, Heidelberg **283**

[35] V. TRANEVA, S. TRANEV (2023) An Elliptic IF Model for Franchisor Selections. In: M. Ganzha, L. Maciaszek, M. Paprzycki, D. Slezak. (eds) "I18th Conference on Computer Science and Intelligence Systems. ". Annals of Computer Science and Information Systems, vol. **37**, pp. 337-343

[36] L. ATANASSOVA (2023) Three de-intuitionistic fuzzification procedures over circular intuitionistic fuzzy sets. *NIFS* **29** (3) 292-297

[37] K. ATANASSOV (1991) "Generalized Nets". World Scientific, Singapore, New Jersey, London

[38] K. ATANASSOV (1987) Generalized index matrices. *Comptes rendus de l'Academie Bulgare des Sciences* **40** (11) 15-18

[39] K. ATANASSOV (2014) "Index Matrices: Towards an Augmented Matrix Calculus". Studies in Computational Intelligence, Springer, Cham **573**