Journal of Theoretical and Applied Mechanics, Sofia, Vol.55 (2025) pp. 105-119
SPECIAL ISSUE

STATIC ANALYSIS OF SUSPENSION SYSTEMS WITH UNIFORMLY DISTRIBUTED VERTICAL LOAD

IHOR RUDKO[1*], BORYS BAKAY[1], ABDULLAH EMIN AKAY[2], VIATCHESLAV LOVEIKIN[3], YURIY ROMASEVYCH[3]

[1]*Ukrainian National Forestry University, Department of Forest Engineering, Lviv, Ukraine*
[2]*Bursa Technical University, Department of Forest Engineering, Bursa, Turkey*
[3]*National University of Life and Environmental Sciences of Ukraine, Department of Machines and Equipment Design, Kyiv, Ukraine*

[Received: 31 July 2024. Accepted: 20 March 2025]

doi: https://doi.org/10.55787/jtams.25.55.1.105

ABSTRACT: Based on classical principles of suspension system calculations, theoretical approaches are proposed for determining the geometric and force parameters of their supporting elements. The calculation of suspension systems with one or multiple spans is proposed to be performed by constructing and solving systems of transcendental equations. For finding solutions to such systems of equations, the graphical-analytical method and the method of successive approximations are used. The paper presents a schematic representation of the diagrams of vertical load, tensions, and sag of the supporting element of the suspension system, the results of numerical modeling (on specific examples), indicates the areas of possible use of the obtained results, and the prospects for further scientific research on the issues under consideration.

KEY WORDS: catenary analysis, suspension systems, flexible element modeling, tension, sag, multi-span structures.

1 INTRODUCTION

Suspension engineering structures (cableways, suspension bridges, etc.) have found wide application in both civil construction and industrial production due to many advantages such as the possibility of laying a route in complex operating conditions, high transport productivity, low energy intensity, and others. The effective operation of suspension engineering structures is influenced by many natural and production factors, therefore, the issues of their design, evaluation of safety factors, and justification of rational technical operation modes are extremely important.

*Corresponding author e-mail: ihor.rudko@nltu.edu.ua

The main element in the suspension engineering structure system is a flexible element (cable, rope, chain, etc.), the shape of the sag curve, geometric and force parameters of which are a key control factor for conducting structural analysis, rational design, force distribution at nodes, etc. [1, 2].

Commonly accepted methods for calculating the operational parameters of flexible elements of suspension engineering structures are the finite element method (a universal method for numerical solution of mechanics problems) and analytical methods [3].

Generally, during the setting of the format of finite elements of suspension engineering structures, a number of simplifications and approximations are made [4, 5], therefore, the obtained calculation models ultimately have limited accuracy. The most common analytical methods for calculating hanging systems in engineering practice are the parabola method and the catenary method [6, 7]. According to the theoretical basis of the parabola method, it is considered that the loading of the suspension structure by its own weight and other loads is distributed uniformly along the length of the span (and not along the length of the sag curve) [1]. In view of this, the parabola method can be considered an approximate calculation method, which can only be applied to calculate the parameters of suspension elements with small sags [6]. The application of the catenary method allows for a more accurate reflection of the shape of the sag of the suspended flexible element [1, 8] and the use of the obtained results in practice, in particular, for optimizing its force and geometric parameters [9].

The use of a discrete model of the supporting element in the calculation of suspension systems can be considered justified only for certain specific tasks [10], since the continuous model of the flexible element is more accurate than the discrete one.

Along with this, for both stationary suspension structures with uniformly distributed vertical load, and in the case of using moving links (cables, carriages, loads, etc.), it is precisely static analysis that is decisive (basic) for their design [11–13].

2 Problem Geometry and Analysis

The theoretical foundations for calculating the supporting elements as the main structures of suspension systems were considered in the works of many scientists (Wyss T. 1956, Zweifel O. 1960, Pestal E. 1961, Kachurin V. 1962, Dukelsky A. 1966, Chung 1987, Sessions J. 1992, Heinimann H. R. 2001, Bont L.G. 2012), who gained wide scientific recognition and currently form the fundamental basis for designing engineering structures and developing computer programs for performing automated calculations [14, 15]. Also, many literary sources provide the corresponding theoretical dependencies and equilibrium equations that characterize the sag shape of the supporting elements, loaded in particular by their own weight or other uniformly distributed vertical loads along the length [2, 13, 16]. However, despite the significant number of available calculation methods for suspension systems [3, 17], it is necessary to pay attention to their different intended purposes and some discrepancies in the obtained results (considering the specifics of mathematical modeling, the obtained theoretical results can differ significantly from each other [18], sometimes up to 70%).

Usually, the input parameters for the problem of establishing the shape, determining the geometric and force parameters of the sag curves of the supporting element, loaded with a uniformly distributed vertical load, are a number of factors, the main of which are:

- characteristics of the longitudinal profile (span length, longitudinal slope of the route, etc.);
- scheme of hanging the supporting element, its physical parameters (breaking strength, linear weight, etc.), payload, etc.;
- natural factors (presence of icing, load from snow, wind, etc. [19].

At the first stage of constructing mathematical models and developing a methodology for calculating the main parameters of the supporting elements in accordance with the main provisions of classical theories of calculation and design of suspension systems, the following assumptions are accepted:

- the flexible supporting element is in the field of gravitational forces and the plane of its sag coincides with the $x - y$ plane, formed, respectively, by the horizontal and vertical axes x and y (Fig. 1);
- the vertical loading (own weight, weight of the winding, coating, etc.) of the flexible supporting element of the suspension structure is uniformly distributed along the arc of its sag curve (if the supporting element of the suspension structure is additionally loaded with many concentrated vertical forces, then their influence in some cases with some approximation can be reduced to the action of a uniformly distributed load [4, 6], but in this case, it is worth assessing the acceptability of a possible error in calculations);
- any section of the flexible supporting element of the suspension structure perceives only tensile forces (at the previous stage of calculation, the influence of the element's rigidity on tension and bending and, accordingly, the change in its shape and cross-sectional area is neglected, since in relative terms it is usually insignificant [7, 11, 20, 21]);
- the ends of the flexible supporting element are rigidly fixed in the supports, and the suspension structure does not have any temperature deformations [11].

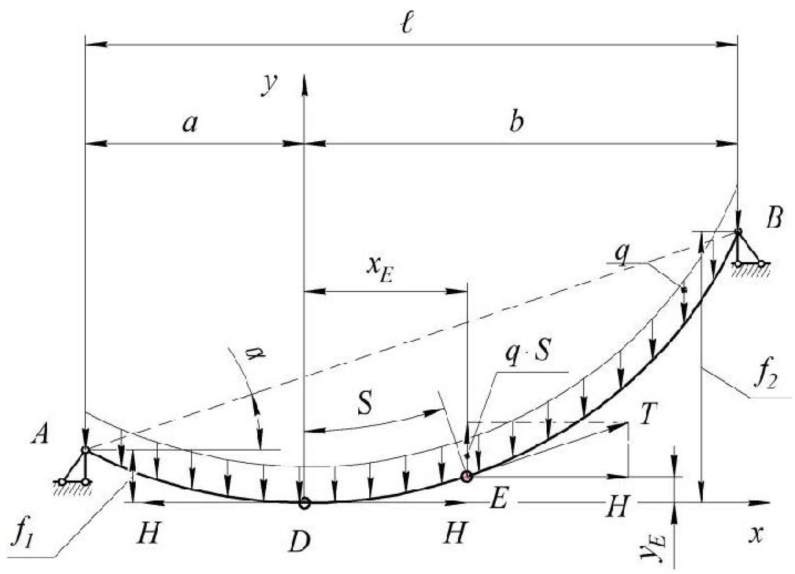

Fig. 1. Schematic diagram of the loaded supporting element in the rectangular coordinate system with the origin at the apex of the sag curve.

To simplify mathematical expressions at the first stage of calculation, it is worthwhile to place the origin of the coordinate system at point D, the apex of the sag curve of the supporting element (Fig. 1).

With the assumptions made, according to the classical theory of designing and calculating suspension systems, the sag shape of the supporting element is described by the catenary [5], the differential equation of which has the form

(1) $$dy/dx = qS/H = S/C_1\,,$$

where q is the intensity of the vertical load uniformly distributed along the curve (linear weight, etc.) [N/m], S is the length of the arc from the origin to an arbitrary point on the arc (for example, point E) [m], H is the projection of the tension of the supporting element onto the horizontal axis x (horizontal component of tension) [N], $C_1 = H/q$ is the catenary parameter [m].

For the established coordinate system, the length of the arc from its origin (point D) to an arbitrary point on the curve with abscissa x (for example, point E) will be [17]

(2) $$S = C_1\,\sinh(x/C_1)\,.$$

In this case, the equation of the sag curve of the supporting element for the coordinate system with the origin at the apex of the catenary (point D) will have the form [19]

$$y = C_1 \left(\cosh(x/C_1) - 1\right) . \tag{3}$$

The magnitude of the tension T of the supporting element at its arbitrary point (for example, point E)

$$T = \sqrt{H^2 + (q\,S)^2} = H \cosh(x/C_1) , \tag{4}$$

where qS is the vertical component of the tension T of the supporting element at its arbitrary point (point E), which is located at a distance S (along the arc) from the origin of the rectangular coordinate system (point D) $\quad [N]$.

From Eq. (3) we obtain that $H \cosh\left(x/C_1\right) = qy + H$.

Taking into account the above, we can conclude that the difference in forces at two arbitrary points on the catenary is equal to the product of the uniformly distributed vertical load and the difference in their ordinates, i.e.

$$T = H + qy . \tag{5}$$

Considering Eq. (3), for distances a and b the following relationships must hold:

$$a = C_1 \,\mathrm{Arcosh}(f_1/C_1 + 1) , \qquad b = C_1 \,\mathrm{Arcosh}(f_2/C_1 + 1) , \tag{6}$$

where f_1 and f_2 are the ordinates that determine the position of the apex of the catenary (point D) relative to the initial (point A) and final (point B) supports [m].

The length of the supporting element is determined as the length of the arc of the explicitly given function $y(x)$:

$$L == \int\limits_{b-\ell}^{b} \sqrt{1 + (y')^2} dx = 2C_1 \sinh(0.5\ell/C_1) \cosh[(b - 0.5\ell)/C_1] . \tag{7}$$

According to Fig. 1

$$f_2 - f_1 = \ell \tan \alpha , \tag{8}$$

where ℓ is the span length (horizontal projection) [m], α is the slope (inclination) of the chord of the span to the horizon [degrees].

From the conditions of the passage of the catenary through the points of attachment (points A and B) in accordance with Eq. (3):

$$f_1 = C_1 \left[\cosh\left((\ell - b)/C_1\right) - 1\right] , \qquad f_2 = C_1 \left(\cosh(b/C_1) - 1\right) . \tag{9}$$

It follows that

$$f_2 - f_1 = 2C_1 \sinh(0.5\ell/C_1) \sinh\left[(b - 0.5\ell)/C_1\right] . \tag{10}$$

Based on Eqs. (7) and (10)

$$L^2 - (f_2 - f_1)^2 = 4C_1^2 \sinh^2(0.5\ell/C_1) . \tag{11}$$

Solving Eq. (11) with respect to L, we obtain

$$L = \sqrt{4C_1^2 \sinh^2(0.5\ell/C_1) + \ell^2 \tan^2\alpha} . \tag{12}$$

Depending on the magnitude of the input data (L, ℓ and α), the desired ratio $C_1 = H/q$ is determined from the given dependency.

By equating the right-hand sides of Eqs. (8) and (10) and performing the necessary transformations, we obtain an expression for calculating the value of b

$$b = \frac{\ell}{2} + C_1 \operatorname{Arsinh}\left[\frac{0.5\ell \tan\alpha}{C_1 \sinh(0.5\ell/C_1)}\right] . \tag{13}$$

Average tension T_{av} of the supporting element within the specified section determined using Eq. (4)

$$T_{\text{av}} = \frac{1}{\ell}\int_{b-\ell}^{b} T(x)dx = \frac{1}{\ell} 2qC_1^2 \sinh(0.5\ell/C_1) \cosh\left((b - 0.5\ell)/C_1\right) . \tag{14}$$

Having established the catenary shape of the supporting element (with a parameter C_1) and the position of the apex of the catenary (values a and f_1), for the convenience of further calculations, we perform a parallel translation of the axes x (upwards by f_1) and y (to the left by a) in order to transition to the coordinate system with the origin at point A (Fig. 2).

In Fig. 2, T_A and T_B are the tensions of the supporting element, respectively, at the lower (point A) and upper (point B) supports [N], T_{A_y} and T_{B_y} are, respectively, the vertical components of the tensions T_A and T_B [N], T_{A_x} and T_{B_x} are, respectively, the horizontal components of the tensions T_A and T_B [N]. If the suspension system is subjected to only a uniformly distributed vertical load along the arc, then we obtain $T_{A_x} = T_{B_x} = H$.

According to the schematic diagram in Fig. 2, the magnitude of the sag, measured from the chord of the span at any point on the curve, is determined by the dependency

$$f = x \tan\alpha - C_1 \left(\cosh\left((x - a)/C_1\right) - 1\right) + f_1 . \tag{15}$$

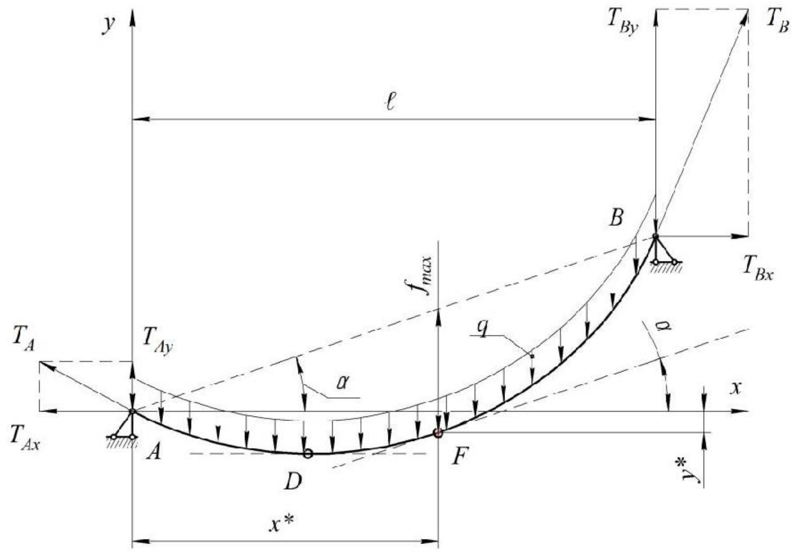

Fig. 2. Schematic diagram of the loaded supporting element in the rectangular coordinate system with the origin at the lower support.

To determine the abscissa x^* (point F), where the sag of the supporting element is maximum ($f = f_{\max}$), we equate the first derivative of the indicated function to zero

(16) $$\frac{df}{dx} = \tan\alpha - \sinh\left(\frac{x^* - a}{C_1}\right) = 0\,, \qquad x^* = a + C_1 \operatorname{Arsinh}(\tan\alpha)\,.$$

Under the condition $x = x^*$, according to Eq. (15) we calculate the value of the maximum sag of the supporting element $f_{\max}$ (Fig. 3), and according to Eq. (17) – the ordinate y^* (point F).

In accordance with the obtained relationships and the possibility of using the stated theoretical principles during engineering calculations, the law of distribution of the horizontal load $q^-(x)$ along the span, the diagrams of tensions $T_y(x)$, $H(x)$ and $T(x)$, as well as the sags of the supporting element $f(x)$, will take the form shown in Fig. 3.

On the diagram (Fig. 3), the positive and negative values of the vertical component of the tension of the supporting element $T_y(x)$ are determined according to the same rule as for the transverse force in the section of the beam (the force is considered positive if the resultant of the external forces to the left of the section is directed from bottom to top, and to the right – vice versa; in the opposite case, the transverse force is considered negative).

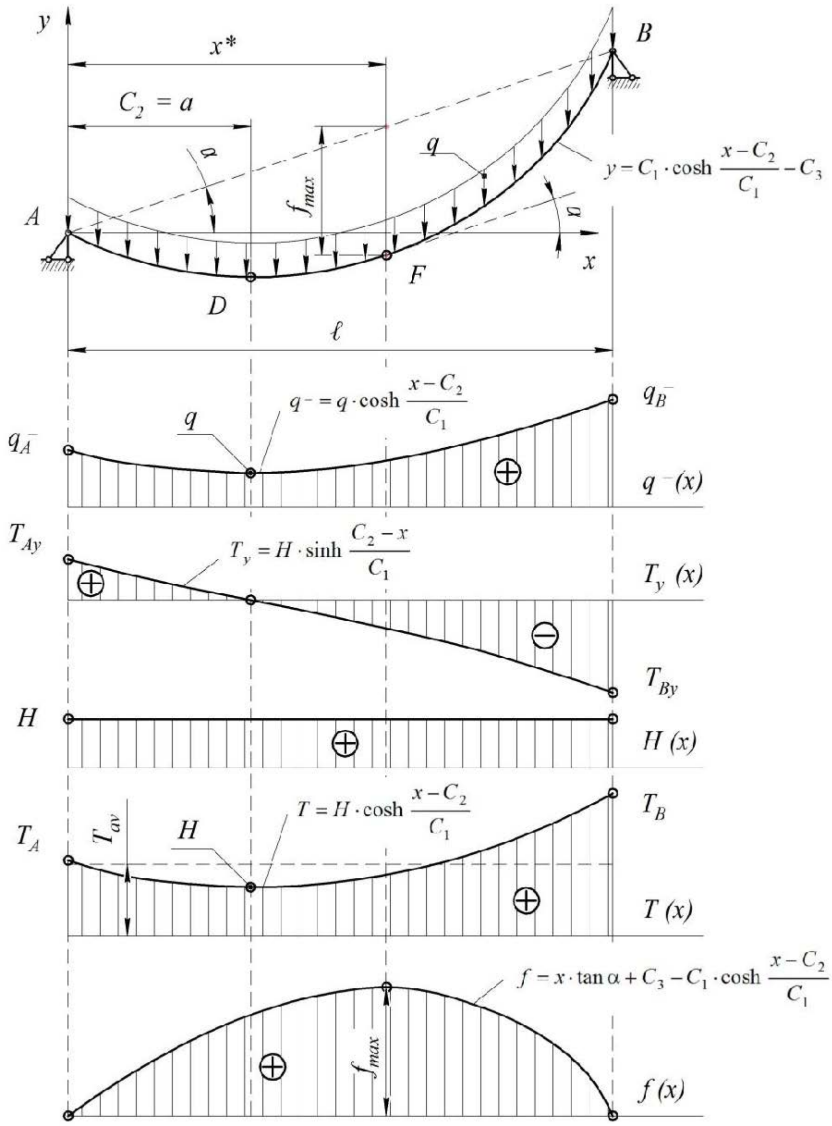

Fig. 3. Schematic representation of vertical load $q^-(x)$, tension $T_y(x)$, $H(x)$, $T(x)$ and sag $f(x)$ diagrams of a supporting element (within one span).

Based on the analysis of Eq. (3) and taking into account the parallel translation of the axes x and y performed, the equation of the sag curve of the supporting element

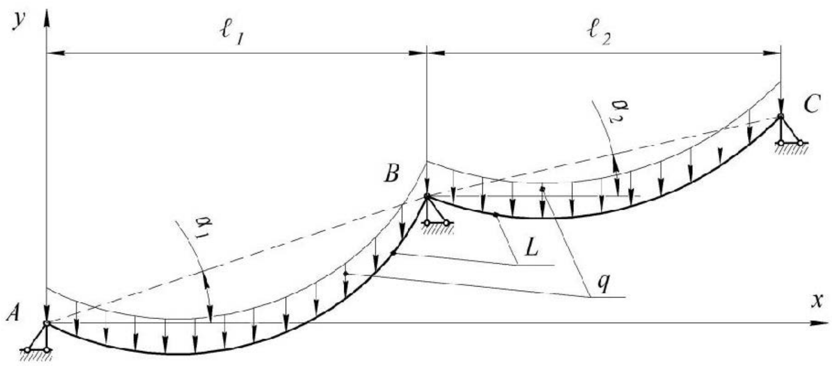

Fig. 4. Design diagram of a multi-span suspension system.

in the coordinate system with the origin at point A (Fig. 3) will have the form [17]

(17) $$y = C_1 \cosh\left((x - C_2)/C_1\right) - C_3\,,$$

where $C_2 = a$ and $C_3 = C_1 + f_1$ are the coefficients of the catenary [m].

The previously mentioned dependencies relate to the calculation of the supporting element, loaded with a uniformly distributed vertical load within one span. However, in engineering practice, multi-span suspension systems are also widely used [16]. Therefore, we will apply the proposed approach to the calculation of a suspension system, for example, with the number of spans $m = 2$ (Fig. 4) and a supporting element, the ends of which are fixed at the lower (point A) and upper (point C) end supports.

It is proposed to calculate this type of multi-span suspension systems using an analogous principle – by constructing and solving systems of transcendental equations (according to the algorithm given in Table 1).

Thus, in accordance with the accepted assumptions and the given algorithm (Table 1), the calculation of the supporting element of a multi-span suspension system with the number of spans m, loaded with a uniformly distributed vertical load q, is reduced to constructing and solving a system of $3m$ transcendental equations that determine the possible values of $3m$ unknown coefficients (parameters) of the catenaries.

In the process of calculating multi-span suspension systems, it is important to note that the obtained solution in the form of defined values of $3m$ unknown coefficients (parameters) of the catenaries for the constructed system of $3m$ nonlinear transcendental equations (Table 1) may not be unique (this is especially characteristic for suspended supporting elements of significant length with a length within the span that exceeds the optimal L_{opt} [9]).

Table 1. Algorithm for constructing systems of transcendental equations for the calculation of supporting elements of multi-span suspension systems

Boundary condition or desired coefficient (parameter)	Notation (i, j and m are positive integers, for a multi-span suspension system the number of spans $m \geq 2$)	Number of equations for the suspension system
	Input data	
Fixation of the ends of the supporting element	$y_1(0) = 0, \quad y_m\Big(\sum_{i=1}^{m} \ell_i\Big) = \sum_{i=1}^{m} (\ell_i \tan\alpha_i)$	2
Placement of the supporting element on intermediate supports	$y_i\Big(\sum_{j=1}^{i} \ell_j\Big) = \sum_{j=1}^{i} (\ell_j \tan\alpha_j),\ i = 1 \ldots m-1$ $y_i\Big(\sum_{j=1}^{i-1} \ell_j\Big) = \sum_{j=1}^{i-1} (\ell_j \tan\alpha_j),\ i = 2 \ldots m$	$2(m-1)$
Equality of tensions of the supporting element at the intermediate supports	$T_i\Big(\sum_{j=1}^{i} \ell_j\Big) = T_{i+1}\Big(\sum_{j=1}^{i} \ell_j\Big),\ i = 1 \ldots m-1$ (respectively for point B and other points defining the position of intermediate supports)	$m-1$
Ensuring the total length of the supporting element	$L = \sum_{i=1}^{m} L_i$	1
	Total	$3m$
	Searched parameters and coefficients	
Number of catenaries	$y_i = C_{1(i)} \cosh \dfrac{x - C_{2(i)}}{C_{1(i)}} - C_{3(i)}, \quad i = 1 \ldots m$	m
Number of unknown coefficients (parameters) – for one catenary	$C_{j(i)},\ j = 1, 2, 3,\ i = 1 \ldots m$	3
	Total	$3m$

3 Numerical Modelling Results (Examples)

Consider an example of calculating the supporting element of a single-span suspension system, for example, for the following input data: $\ell = 350$ m, $\alpha = -30°$, $q = 25$ N/m, $L = 425$ m.

To determine the value of the parameter $C_1 = H/q$ from the transcendental equa-

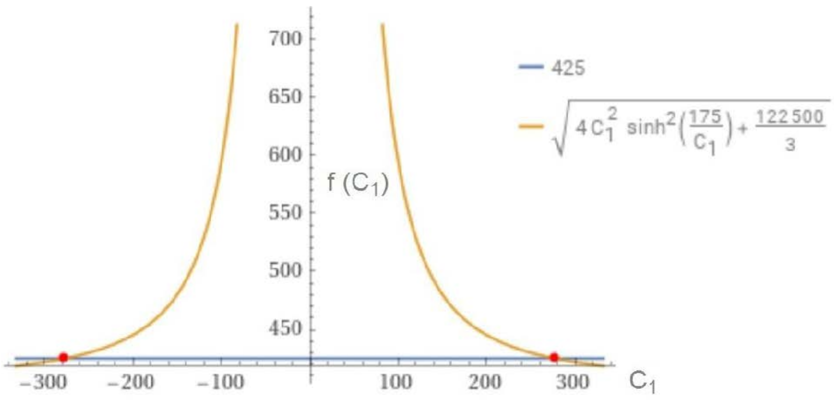

Fig. 5. Determination of the catenary parameter $C_1 \approx 276.22891$ m a using the graphical method in the WolframAlpha computational knowledge engine (wolframalpha.com).

tion (12), the method of successive approximations (iterative method [2,5,20]) or the graphical method using computational algorithms of specialized software products (Fig. 5) are used.

In the following, the negative value of the solution of the transcendental equation (12) $C_1 \approx -276.22891$ m will not be considered, since in this case the problem loses its engineering relevance, because according to the condition of the problem, the supporting element of the suspension structure perceives only tensile forces.

Then, in accordance with Eqs. (1) – $H \approx 6905.723$ N; (13) – $b \approx 32.160$ m; (9) – $f_1 \approx 203.947$ m and $f_2 \approx 1.874$ m; (4) – $T_A \approx 12004.393$ N and $T_B \approx 6952.578$ N; (14) – $T_{\text{av}} \approx 8385.520$ N; (6) and (17) – $C_2 = a \approx 317.84020$ m and $C_3 \approx 480.17571$ m.

Having calculated the value of the parameter and the coefficients of the catenary, in accordance with the sample (Fig. 3), we construct the sag curve of the flexible supporting element, the diagrams of its tension and sag (Fig. 6).

According to the Eqs. (15) and (16), the sag of the supporting element is maximum at $x^* \approx 166.106$ m (the deviation from the middle of the span $\ell 2 = 175$ m is -8.894 m or -5%), and the value of the maximum sag of the supporting element is $f_{\max} \approx 65.313$ m.

The parameters of the supporting element of a multi-span suspension system are determined by a similar principle.

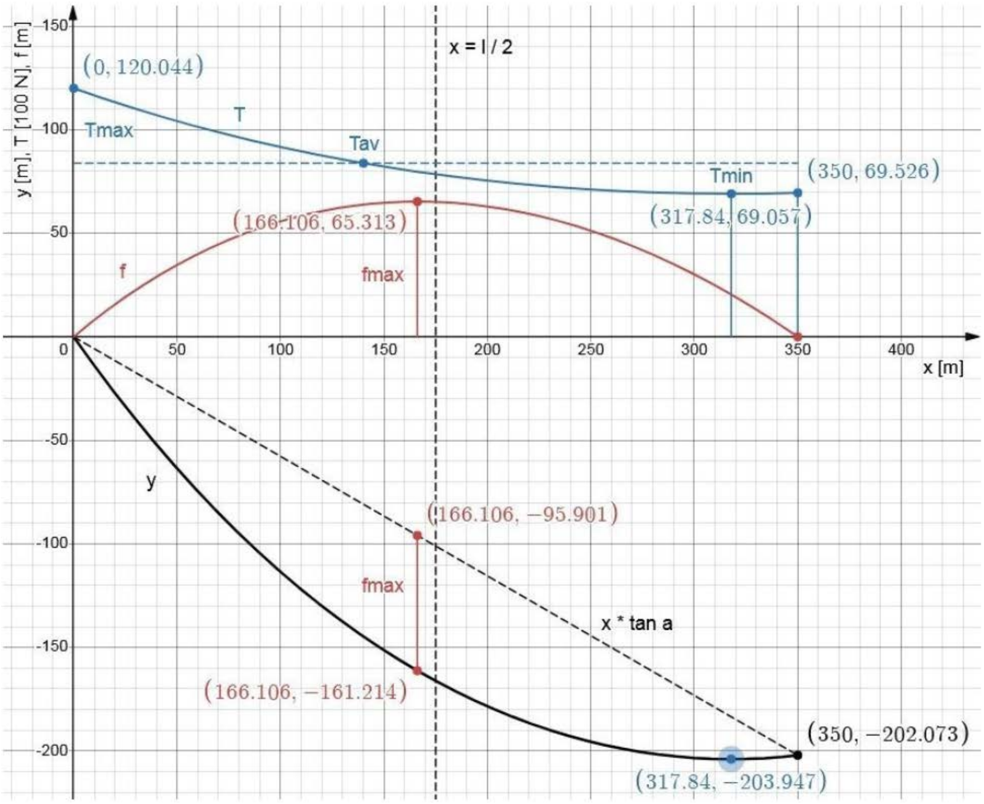

Fig. 6. Sag curve y, diagrams of tension T and sag f of the flexible supporting element with its ends fixed rigidly in supports within one span (length $\ell = 350$ m).

4 Conclusions

This scientific article discusses the issues related to improving models and theoretical principles for calculating arbitrarily tensioned the supporting elements of suspension systems. A general approach to modeling and calculating the geometric and force parameters of the supporting elements of suspension installations with a uniformly distributed vertical load is proposed. The shape of the sag curves of the supporting elements is presented in the form of catenaries. An algorithm has been developed for constructing systems of transcendental equations to determine the parameters and coefficients of the catenaries, which are rationally solved using the graphical-analytical method and the method of successive approximations (the number of calculations is insignificant, and the accuracy of the result is high). The possibility of practical application of the developed algorithm is demonstrated on specific example (calculation of the supporting element of single-span suspension system was carried out).

The presented theoretical principles for modeling suspension systems generally do not contradict the fundamental principles of classical calculation methods and, at the same time, expand the boundaries of their practical application regarding the design, operation, and compliance with the requirements of engineering safety of suspended engineering structures. In general, the results of the study can be applied both in the field of design of technological and transport equipment, and in related fields of mechanical engineering, which involve calculations of flexible supporting elements (chains, cables, ropes, strings, cords, etc.), which are loaded with a uniformly distributed vertical load along the length (own weight or other load).

As a result of the analysis of the dependencies presented in the paper, the following conclusions can be drawn regarding the prospects for further research:

- the presented theoretical principles should be used as the basis for improving the methods of static calculation of certain types of suspension engineering structures (two-dimensional single-span and multi-span, three-dimensional coupled cable structures [12, 13], etc.), as well as dynamic calculation of flexible supporting elements (as continuous models);
- in the process of forming applied engineering calculation methods, it is necessary to highlight the main factors influencing the operation of the supporting elements of specific suspension systems (in order to take into account their design features, operating conditions, and adequately reflect this influence in mathematical models);
- if necessary to take into account, for example, the flexibility of the supports, elastic deformation or temperature change of the supporting element [19], friction forces that arise during its movement through the intermediate supports, the influence of natural and other factors, it is necessary to make changes to the corresponding boundary conditions and equilibrium equations (the proposed approach provides for this possibility, in particular, for supporting elements made of heterogeneous materials [21], for example, certain types of cables that do not have a defined yield point and practically do not obey the fundamental law of elasticity of R. Hooke).

REFERENCES

[1] T. Jiang, Y. Yu, X. Zhang, W. Huang (2020) Comparative analysis of calculation methods for Cable Curve of Landscape Suspension Bridges. *Journal of Physics: Conference Series. 2nd International Conference on Computer Modeling, Simulation and Algorithm* **1624**(4) 042043.

[2] J. Qin, Y. Ba, Y. Ding, J. Bai, H. Zhang (2015) Newton Iteration Method for Analysis of Suspension Cable. In: *International Conference on Modeling, Simulation and Applied Mathematics*, Atlantis Press, 350-354.

[3] W. ZHANG, G. TIAN, Y. CHEN (2024) Evolution of Suspension Bridge Structural Systems, Design Theories, and Shape-Finding Methods: A Literature Survey. *Journal of Traffic and Transportation Engineering (English Edition)* **11**(2) 225-244.

[4] J. KOWAL, J. SNAMINA, A. PODSIADLO, J. KONIECZNY (2008) Static and dynamic analysis of the cableway. *Archive of Mechanical Engineering* **4** 357-368

[5] J. QIN, J. CHEN, L. QIAO, J. WAN, Y. XIA (2016) Catenary analysis and calculation method of track rope of cargo cableway with multiple loads. *MATEC Web of Conferences* **82** 01008.

[6] G. LIU, Y. WANG (2018). Study on the Calculation Theory of the Cable Curve for Suspension Bridge. *IOP Conference Series: Materials Science and Engineering* **394**(3) 032092.

[7] W. REN, M. HUANG, W. HU (2008) A parabolic cable element for static analysis of cable structures. *Engineering Computations (Swansea, Wales)* **25**(4) 366-384.

[8] G. TIAN, W. ZHANG, J. CHANG, Z. LIU (2023) Analytical study on cable shape and its lateral and vertical sags for earth-anchored suspension bridges with spatial cables. *Structural Engineering and Mechanics* **87**(3) 255-272.

[9] I. RUDKO, B. BAKAY, L. G. BONT, Y. ROMASEVYCH, S. HORZOV (2022) Calculation of optimal length for skyline. *Bulletin of the Transilvania University of Brasov, Series II: Forestry, Wood Industry, Agricultural Food Engineering* **64**(15) 79-90.

[10] D. AGMON, H. YIZHAQ (2020) A new solution of the discrete catenary problem. *European Journal of Physics* **41**(2) 025002.

[11] S. DUPIRE, F. BOURRIER, F. BERGER (2016) Predicting load path and tensile forces during cable yarding operations on steep terrain. *Journal of Forest Research* **21**(1) 1-14.

[12] M. SUCH, J.R. JIMENEZ-OCTAVIO, A. CARNICERO, O. LOPEZ-GARCIA (2009) An approach based on the catenary equation to deal with static analysis of three dimensional cable structures. *Engineering structures* **31**(9) 2162-2170.

[13] L. GRECO, N. IMPOLLONIA, M. CUOMO (2014) A procedure for the static analysis of cable structures following elastic catenary theory. *International Journal of Solids and Structures* **51**(7-8) 1521-1533.

[14] O. MOLOGNI, L. MARCHI, C.K. LYONS, S. GRIGOLATO, R. CAVALLI, D. RÖSER (2021) Skyline tensile forces in cable logging: Field observations vs. software calculations. *Croatian Journal of Forest Engineering* **42**(2) 227-243.

[15] L.G. BONT, P.E. MOLL, L. RAMSTEIN, F. FRUTIG, H.R. HEINIMANN, J. SCHWEIER (2022) SEILAPLAN: a QGIS Plugin for Cable Road Layout Design. *Croatian Journal of Forest Engineering* **43**(2) 241-255.

[16] L. BONT, H.R. HEINIMANN (2012) Optimum geometric layout of a single cable road. *European Journal of Forest Research* **131**(5) 1439-1448.

[17] C. KNOBLOCH, L.G. BONT (2021) A new method to compute mechanical properties of a standing skyline for cable yarding. *PLoS ONE* **16**(8) e0256374.

[18] H.R. HEINIMANN (2001) Approaches to the design of forest cable systems. In: *Proceedings of Workshop New trends in wood harvesting with cable systems for sustainable forest management in the mountains*, June 18-24, Ossiach, Austria; Available at: https://www.fao.org/4/y9351e/Y9351E00.htm. Accessed on: 29 July, 2024.

[19] X. DONG (2016) Analytic Method to Calculate and Characterize the Sag and Tension of Overhead Lines. *IEEE Transactions on Power Delivery* **31**(5) 2064-2071.

[20] J. VASEK, O. SUCHARDA (2014) Numerical analysis of gravity and parabolic catenaries. *International Journal of Mathematical Models and Methods in Applied Sciences* **8**(1) 156-164; Available at: https://www.naun.org/main/NAUN/ijmmas/2014/a302001-314.pdf. Accessed on: 29 July, 2024.

[21] E. COARITA, L. FLORES (2015) Nonlinear Analysis of Structures Cable-Truss. *IACSIT International Journal of Engineering and Technology* **7**(3) 160-169.