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NUMERICAL STUDY OF FGM SANDWICHES PLATE UNDER THE EFFECT OF THERMO-MECHANICAL LOADS

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ABSTRACT: This article presents a study on the behavior of rectangular functionally graded plates under thermo-mechanical bending conditions. The study employs a finite element method to analyze these plates, consisting of functionally graded face sheets and core. Validating the proposed model's accuracy through comparisons with existing literature, the investigation explores the effects of key parameters thermal load, geometric factors, and volume fraction distribution on thermo-mechanical bending behavior. The study includes a thorough parametric analysis to identify significant factors impacting normal stresses and deflection of functionally graded plates. The findings offer valuable insights for designing functionally graded plates subjected to combined thermal and mechanical loads. With its simplicity and potential for future advancements, the suggested method proves highly suitable for addressing these problems.

KEY WORDS: Composite materials, functionally graded materials, thermo-mechanical bending, Finite element, Sandwich plate.

1 INTRODUCTION

The study of functionally graded material (FGM) sandwich plates under thermo-mechanical loads is an intricate and significant field of research, focusing on their behavior under combined thermal and mechanical stresses. This area is particularly relevant for advanced engineering applications, including aerospace, structural systems, and other fields where bending, buckling, and dynamic responses are critical. Analytical and numerical approaches, such as the finite element method (FEM), are

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extensively employed to model and analyze these responses, providing critical insights into the effects of material gradation, thermal environments, and geometric configurations.

FGMs are a unique class of composite materials characterized by a gradual variation in composition and structure, leading to a progressive change in properties [1]. This gradient imparts superior performance compared to conventional composites, combining the advantages of ceramics, such as wear and oxidation resistance, with the benefits of metals, including machinability, hardness, and thermal resistance. Consequently, FGMs have found diverse applications in fields such as medicine [2], the chemical industry [3], aeronautics [4], aerospace [5], marine engineering [6], and civil engineering [7–9].

Numerous analytical and numerical methods have been developed to explore the behavior of FGM plates. Methods such as Navier's solution for shear deformation theories applied to simply supported plates [10], Levy's approach [11, 12], the differential quadrature method [13], the Rayleigh-Ritz method [14], and FEM are particularly prevalent in free vibration analysis.

Several notable contributions have enriched the understanding of FGM behavior. A three-dimensional finite element model based on Hamilton's principle has been used to analyze the free vibration of periodic composite plates [15]. A novel layerwise finite element formulation employing the mixed-least-squares approach was introduced to investigate both static and free vibration behaviors of periodic composite plates [16, 17]. Isoparametric elements combined with Mindlin plate theory were applied for analyzing isotropic and composite plates [18], while a high-order finite element method was employed to study the dynamic response of soft-core sandwich plates [19].

For FGM sandwich plates, research has addressed thermal loads and the influence of geometric and material parameters [23–25]. Shear-deformable plate theories have been proposed to analyze thermo-elastic bending [26]. The coupled vibration characteristics of porous annular and circular plates under magneto-electro-elastic conditions have been explored using higher-order shear deformation theory (HSDT) and FEM in polar coordinates [27]. The thermal vibration behavior of FG sandwich-shell structures has also been studied using isoparametric quadrilateral-Lagrangian models and higher-order theories [28].

Key findings indicate that material gradation significantly influences the bending response, which can vary with uni-directional or bi-directional property changes. Temperature-dependent material properties and gradation exponents affect deflection and stress distribution, as shown in studies employing inverse trigonometric shear deformation theory (ITSDT) [28]. Refined shear theories have demonstrated the impact of various parameters on bending behavior under thermo-mechanical loads [29].

The dynamic response of FGM nanoplates in thermal environments highlights the role of material gradation and thermal effects on natural frequencies [30]. Moreover, porosity and material grading indices influence thermal buckling behavior, with different porosity distribution functions affecting buckling loads [31]. Nonlinear static behaviors of bi-directional FGM plates have been analyzed using higher-order FEM, revealing the influence of temperature-dependent material properties and geometric nonlinearity on static responses [32].

The optimization of FGM structures is critically dependent on material gradation, thermal loads, and geometric factors. Variations in material properties significantly affect deflection and stress responses, with distinct gradient profiles producing varied mechanical behaviors under loading [33]. Additionally, temperature-dependent characteristics influence deflection and stress distributions, while geometric factors, such as aspect ratio and layer thickness, play essential roles in the bending response, improving performance under combined loads [34]. FEM remains a cornerstone for modeling the complex interactions within FGM plates, facilitating detailed parametric studies and validation against existing literature [35].

Advanced shear deformation theories like ITSDT offer more accurate modeling of displacement functions, simplifying the analysis. However, limitations such as computational costs and the need for extensive validation remain challenges for future research, which must balance theoretical models and practical applications.

The novelty of this study lies in the development of a finite element model to analyze the thermo-mechanical behavior of functionally graded sandwich plates composed of functionally graded face sheets and core. The study incorporates a comprehensive parametric analysis to evaluate the combined effects of thermal and mechanical loads, geometric factors, and material properties on the bending behavior. This work offers new insights into the design of functionally graded plates subjected to thermo-mechanical loads, providing a simple yet robust method for future advancements in this field.

2 Theoretical Formulation

In the context of the functionally graded material (FGM) plate, let's consider a rectangular plate with dimensions h, a, and b for thickness, length, and width, respectively, as depicted in Fig. 1. The Cartesian coordinate system XYZ is employed, where the plane $z = 0$ represents the mid-area of the rectangular FGM plate. This plate consists of three layers: a FGM core sandwiched between two face sheets. At the upper surface of the sandwich plate, a transverse mechanical force is applied, while a variable thermal load is distributed throughout the thickness. The upper face sheet exhibits alternating zones of ceramic-rich composition and metal-rich composition, while the bottom face sheet exhibits a reverse arrangement with metal-rich and ceramic-rich zones, as illustrated in Fig. 1.

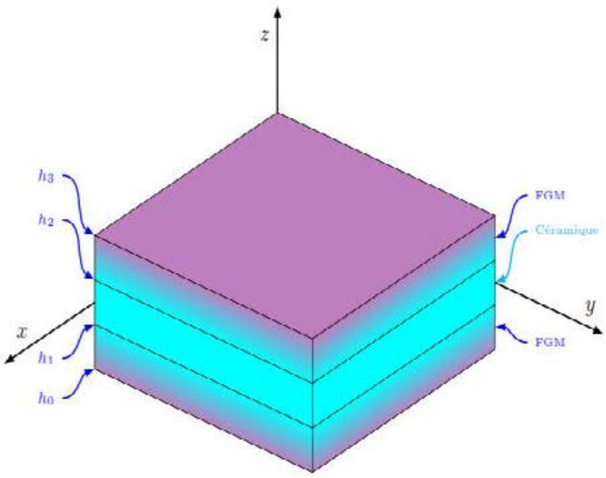


Fig. 1. Rectangular sandwich FGM plate

A power-law function governs the face sheets' volume portion via the thickness

$$V^{(1)} = \left(\frac{z - h_0}{h_1 - h_0}\right)^p \qquad z \in [h_0,\, h_1]\,, \tag{1}$$

$$V^{(3)} = \left(\frac{z - h_3}{h_2 - h_3}\right)^p \qquad z \in [h_2,\, h_3]\,, \tag{2}$$

where $V^{(n)}$ represents the volume fraction of the n-th layer ($n = 1, 3$). Also, the sandwich core's volume fraction is indicated as

$$V^{(2)} = 1 \qquad z \in [h_1,\, h_2]\,, \tag{3}$$

p represents the power index material and accepts only values that exceed or match zero.

The thermal expansion factors, Poisson's ratio, and Young's modulus may be defined as

$$f^{(n)}(z) = f_m \exp\left(\beta V^{(n)}\right) \qquad \text{for} \quad n = 1, 2, 3\,, \tag{4}$$

$$\beta = \ln\left(\frac{f_c}{f_m}\right) \tag{5}$$

"c" and "m" represent ceramic and metal, respectively.

This formula (4) accounts for the graded transition of material properties within each layer of the sandwich structure, enabling precise modeling of behavior under thermal or mechanical loads.

In the plate, an arbitrary point's displacements may be defined as [20]

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0(x,y)}{\partial x} + \Psi(z)\theta_x(xy)\,, \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0(x,y)}{\partial y} + \Psi(z)\theta_y(xy)\,, \\ w(x,y,z) &= w_0(x,y)\,, \end{aligned} \tag{6}$$

where u, v, and w represent the x, y, and z displacements respectively; u_0, $v_{0,}$ and w_0 denote midplane displacements. θ_x and θ_y are rotations in the yz and xz planes.

$\Psi\,(z)$ signifies the form function that governs the thickness-direction transverse shear strain and stress distribution and can be expressed as

$$\Psi(z) = z\left(1 - \frac{4z^2}{3h^2}\right)\,. \tag{7}$$

Considering $\Psi(z) = 0$, we obtain the displacement of the classical-thin-plate-theory (CPT). And the displacement of the first-order-shear-deformation-plate-theory (FSDPT) resulting from setting $\Psi(z) = z$.

The transverse displacement w might be stated as

$$w(x,y,z) = w_b(x,y) + w_s(x,y)\,, \qquad \theta_x = \frac{\partial \omega_s}{\partial x}\,, \qquad \theta_y {=} \frac{\partial \omega_s}{\partial y}\,, \tag{8}$$

where w_b and w_s represent the bending component and the shear component of w respectively.

In the current refined plate theory, the displacement is defined as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial \omega_b}{\partial x} + f(z)\frac{\partial \omega_s}{\partial x}\,, \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial \omega_b}{\partial y} + f(z)\frac{\partial \omega_s}{\partial y}\,, \\ w(x,y,z) &= w_b(x,y) + w_s(x,y)\,, \\ f(z) &= z - \Psi(z) \end{aligned} \tag{9}$$

According to equation (9), the strain expressions are as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s[3pt] \end{Bmatrix}\;; \quad \varepsilon_z = 0\,, \tag{10}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = [1 - f'(z)] \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{11}$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \tag{12}$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \tag{13}$$

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = - \begin{Bmatrix} \dfrac{\partial^2 w_b}{\partial x^2} \\ \dfrac{\partial^2 w_b}{\partial y^2} \\ 2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \tag{14}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = - \begin{Bmatrix} \dfrac{\partial^2 w_s}{\partial x^2} \\ \dfrac{\partial^2 w_s}{\partial y^2} \\ 2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \tag{15}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix}. \tag{16}$$

The stress-strain relationships for the FG sandwich plates may be written as follows, taking into account the thermal impact for the n-th layer: $(\sigma_x \sigma_y, \tau_{yz}, \tau_{xz}, \tau_{xy})$ and $(\varepsilon_x, \varepsilon_Y, \gamma_{yz}, \gamma_{xz}, \gamma_{xy})$:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}^{(n)} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}^{(n)} \begin{Bmatrix} \varepsilon_x - \alpha T \\ \varepsilon_y - \alpha T \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(n)}, \tag{17}$$

$$c_{11}^{(n)} = c_{22}^{(n)} = \frac{E^{(n)}z}{1-\left(\mu^{(n)}\right)^2}\,, \tag{18}$$

$$c_{44}^{(n)} = c_{55}^{(n)} = c_{66}^{(n)} = \frac{E^{(n)}(z)}{2\left(1+\mu^{(n)}\right)}\,, \tag{19}$$

$$c_{12}^{(n)} = \mu^{(n)} c_{11}^{(n)}\,. \tag{20}$$

The generalized temperature field T is defined as

$$T(x,y,z) = T_1(x,y) + \frac{h}{T_2}(x,y) + \frac{\Psi(z)}{h} T_3(x,y)\,, \tag{21}$$

where T_1, T_2 , and T_3 represent thermal loads.

The thermo-mechanical bending issue of the FGM sandwich plate uses the theory of virtual work. The FGM sandwich plate's total potential energy is provided by

$$U = \frac{1}{2}\int\limits_V \Big[\sigma_x^{(n)}(\varepsilon_x - \alpha T)^{(n)} + \sigma_y^{(n)}(\varepsilon_y - \alpha T)^{(n)} + \tau_{xy}^{(n)}\gamma_{xy}^{(n)} + \tau_{xz}^{(n)}\gamma_{xz}^{(n)} + \tau_{yz}^{(n)}\gamma_{yz}^{(n)}\Big] dV - \int\limits_\Omega qw d\Omega\,. \tag{22}$$

The virtual work concept for the current issue can be stated as follows:

$$\int\limits_\Omega \Big(N_x \delta\varepsilon_x^{(0)} + N_y \delta\varepsilon_y^{(0)} + N_{xy}\delta\gamma_{xy}^{(0)} + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + Q_{yz}^s \delta\gamma_{yz}^{(0)} + Q_{xz}^s \delta\gamma_{xz}^{(0)}\Big) d\Omega - \int\limits_\Omega q(\delta w_b + \delta w_b) d\Omega = 0\,, \tag{23}$$

where M, N, and Q are the stress resultants.

All calculations done [20], the following operator equation is obtained

$$[\mathbf{K}]\,\{\mathbf{U}\,\mathbf{V}\,\mathbf{W}_b\,\mathbf{W}_s\}^{\mathrm{T}} = \{\mathbf{P}\}\;, \tag{24}$$

where the elements of the symmetric matrix $[\mathbf{K}]$ presented as

$$K_{11} = A_{11}\lambda^2 + A_{66}\mu^2\,,$$
$$K_{12} = \lambda\mu(A_{12} + A_{66})\,,$$
$$K_{13} = -\lambda[B_{12}\lambda^2 + (B_{12} + 2B_{66})\mu^2]\,,$$

$$
\begin{aligned}
K_{14} &= -\lambda[C_{11}\lambda^2 + (C_{12} + 2C_{66})\mu^2]\,, \\
K_{22} &= A_{66}\lambda^2 + A_{22}\mu^2\,, \\
K_{23} &= -\mu[B_{22}\mu^2 + (B_{12} + 2B_{66})\lambda^2]\,, \\
K_{24} &= -\mu[C_{22}\mu^2 + (C_{12} + 2C_{66})\lambda^2]\,, \\
K_{33} &= D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4\,, \\
K_{34} &= F_{11}\lambda^4 + 2(F_{12} + 2F_{66})\lambda^2\mu^2 + F_{22}\mu^4\,, \\
K_{44} &= H_{11}\lambda^4 + 2(H_{12} + 2H_{66})\lambda^2\mu^2 + H_{22}\mu^4 + J_{55}\lambda^2 + J_{44}\mu^2\,.
\end{aligned}
\tag{25}
$$

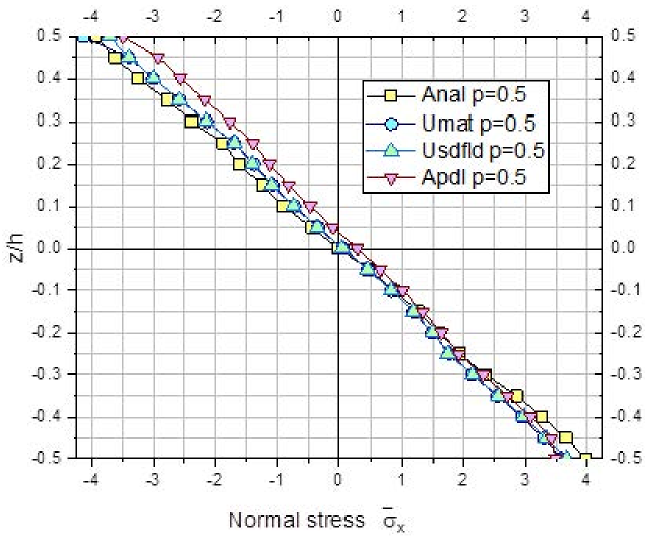
3 Validation

To evaluate the accuracy of the numerical model, we investigated the thermo-mechanical bending behavior of functionally graded material (FGM) plates. The numerical simulations were conducted using Abaqus and Ansys Apdl software. Specifically, we implemented the umat and usdfld subroutine options in Abaqus. The validation was performed by comparing these results against the analytical solution for (1-2-1) sandwich plates, as reported in the work of Li et al. [20].

The analytical model used by Li et al. provides a theoretical framework for evaluating normal stress distributions in FGM plates with homogeneous face sheets and a graded core. This solution is derived by solving the governing equilibrium equations while considering the material gradation across the thickness based on a power-law distribution.

Figure 2 presents the variation of normal stress (σx) along the plate thickness (z/h) for three different gradation parameters ($p = 0.5$, $p = 2.5$, and $p = 3.5$).

(a)



(b)

(c)

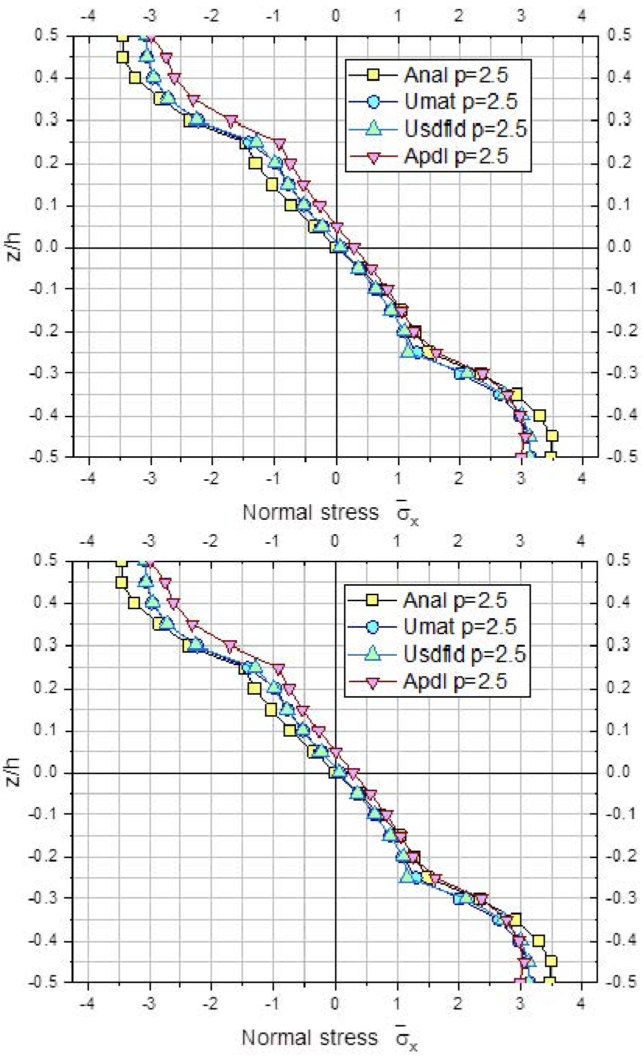


Fig. 2. Normal stress variation along sandwich plates' thickness for various p values with homogenous face sheets and FGM cores: (a) $p = 0.5$; (b) $p = 2.5$; (c) $p = 3.5$.

The parameter p controls the material property variation through the plate's thickness. Lower values of p correspond to a smoother transition between the properties of the ceramic and metal phases, while higher p values indicate a steeper gradient. The material gradation is a critical factor influencing the stress distribution in the FGM core.

Figure 2(a) shows the normal stress distribution for $p = 0.5$. The stress profile is nearly linear, reflecting a gradual variation in the material properties across the thickness. The analytical solution, represented by black squares, aligns closely with the results from the umat and usdfld options in Abaqus, depicted by red triangles and blue diamonds, respectively. In contrast, the results obtained from Ansys Apdl (orange lines) deviate noticeably, particularly near the face sheets.

Figure 2(b) depicts the normal stress distribution for $p = 2.5$. As the gradation parameter increases, the stress profile becomes more nonlinear, with noticeable curvature near the face sheets. Similar to the $p = 0.5$ case, the Abaqus results (umat and usdfld) exhibit strong agreement with the analytical solution, whereas the Apdl results demonstrate a higher deviation, especially in regions near the interfaces between the face sheets and the core.

Figure 2(c) illustrates the stress distribution for $p = 3.5$. For this case, the stress profile exhibits even greater nonlinearity, highlighting the steeper material property transition. The Abaqus results remain consistent with the analytical solution, with only minor discrepancies observed. However, the Apdl results show significant divergence, particularly in regions of higher stress gradients.

3.1 Accuracy of numerical models

The comparison confirms that the Abaqus software, utilizing the umat and usdfld options, provides results that closely match the analytical benchmark, with a deviation of only 0.9%. This highlights the robustness and accuracy of the Abaqus implementation in capturing the stress variation in FGM plates. On the other hand, the Ansys Apdl results deviate by over 20%, particularly for higher pp values, which may be attributed to limitations in its formulation or numerical precision when handling steep gradients.

The computations were performed on an Intel(R) Core(TM) i5-9500 CPU @ 3.00 GHz with 16 GB of RAM, using C3D20R elements (20-node quadratic bricks with reduced integration) and mesh refinement of 2–30 divisions in the thickness (z-axis) and 1 in the in-plane directions (x, y). While the proposed model accurately captures complex displacement fields in FGMs, future work could address its limitations by integrating the UEL subroutine in Abaqus for tailored displacement fields and custom mesh elements. The model could potentially be extended to dynamic loading scenarios in the frequency domain, with appropriate modifications. Compared to recent studies, the approach demonstrates innovation in combining analytical and numerical methods for improved accuracy in stress and displacement predictions for sandwich plates with FGM face sheets.

Based on the findings, Abaqus was selected as the primary tool for subsequent analyses due to its superior accuracy and consistency with the analytical results. This validation establishes confidence in the numerical model for evaluating the thermo-mechanical response of FGM plates under various loading and boundary conditions.

4 Parametric Study

A comprehensive parametric analysis will be performed to assess the effects of major parameters on the normal stress and deflection. This analysis will employ the single variable technique, whereby one parameter is varied while keeping the others constant. In this section, we will focus on the results obtained for a 1-2-1 sandwich plate, but the discussion is applicable to other sandwich plates as well.

4.1 Effect of temperature

The effect of temperature on the behavior of functionally graded material (FGM) sandwich plates has examined, while keeping the power-law exponent (p) constant at 1.5. The remaining parameters will be maintained as defined in the previous section. Through this parametric analysis, we aim to gain insights into the influence of temperature on the normal stress distribution and displacement behavior of FGM sandwich plates.

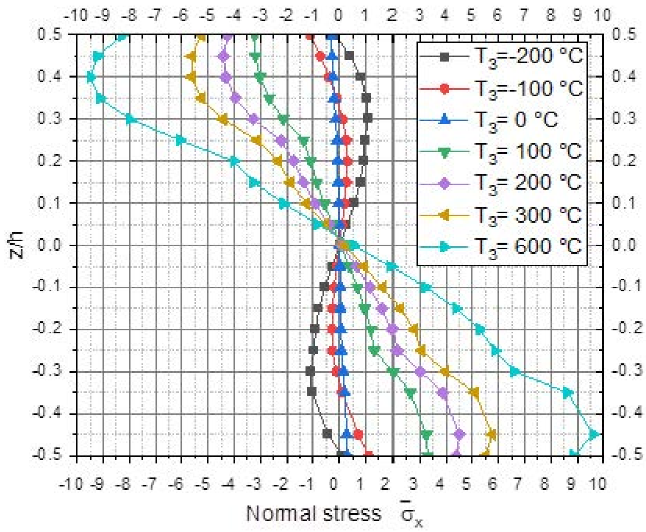


Fig. 3. The behavior of normal stress via the thickness changing the temperature T3.

Figure 3 illustrates the variation of the normal stress along the thickness of the sandwich plate as the temperature (t3) changes. It can be observed that, for negative temperatures, the lower face sheet experiences the minimum compressive stress, while the upper face sheet undergoes the maximum tensile stress. As expected, the normal stress increases with rising temperatures.

Additionally, Fig. 4 displays the displacement through the thickness of the plate as the temperature (T3) is changed.

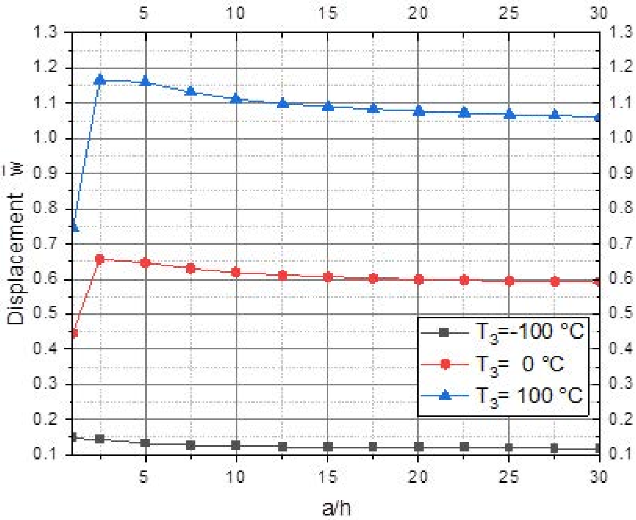


Fig. 4. Variation of the displacement with a/h ratio for different temperature changes (t3 $= -100°$, t3 $= 0°$ and t3 $= 100°$).

It is evident that the displacement increases with an increasing ratio of a/h. Furthermore, for a/h ratios exceeding 5, the displacement remains unchanged. Moreover, the increase in temperature leads to an amplification of the displacement.

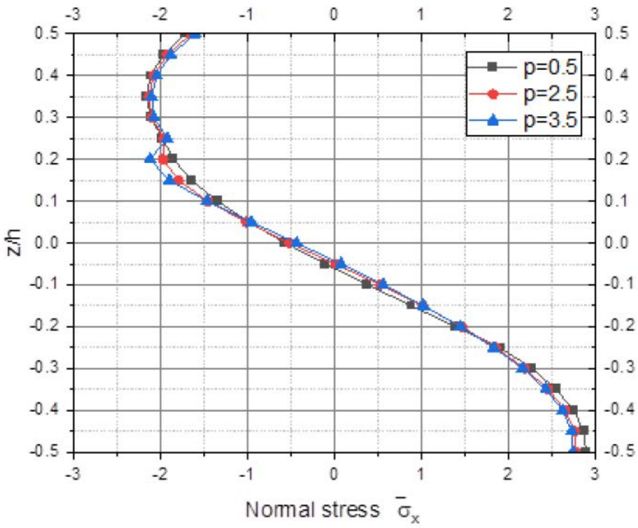


Fig. 5. Variation of normal stress through thickness for different material indices ($p = 0.5$, $p = 2$, and $p = 3.5$).

4.2 Effect of p

In this part of the analysis, we will investigate the effect of the power-law exponent (p) on the behavior of functionally graded material (FGM) sandwich plates. The temperature parameters (t2 and t3) will be set to 100, and the ratio of a/h will be fixed at 10. By analyzing the influence of the power-law exponent (p) on the normal stress and displacement of FGM sandwich plates, we aim to gain a deeper understanding of how this parameter impacts the mechanical response of the plates under the specified conditions.

Figure 5 presents the behavior of the normal stress along the thickness of the sandwich plate for different values of the power-law exponent (p). It can be observed that as the value of p increases, the normal stress generally decreases. This indicates that a higher power-law exponent leads to a reduction in the normal stress distribution throughout the plate.

On the other hand, Figure 6 reveals that there is no significant effect of the power-law exponent (p) on the displacement behavior of the plate. The displacement remains relatively consistent across different values of p. We keep t2 $=$ t3 $= 100$ and $a/h = 10$.

Figure 5 shows the behavior of the normal stress via the thickness varying p values. Typically, the normal stress decreases as the coefficient p is raised. While there is no significant effect of p in the displacement as evident in Fig. 6.

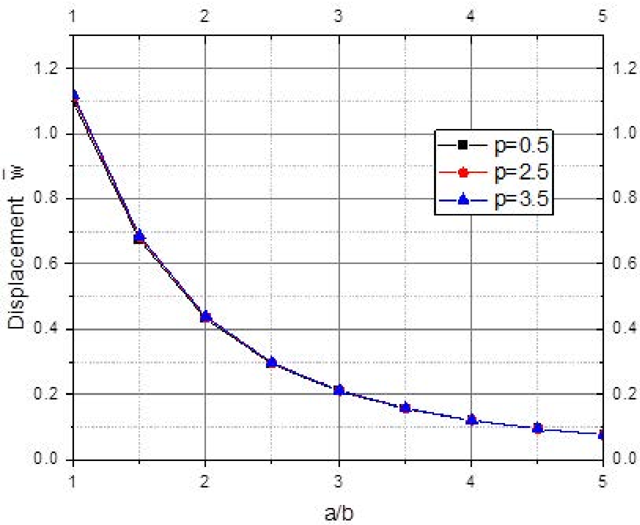


Fig. 6. Variation of displacement with a/b for different material indices ($p = 0.5$, $p = 2$, and $p = 3.5$).

5 Conclusion

In conclusion, this study reports a comprehensive analysis of the thermo-mechanical bending behavior of a rectangular functionally-graded-plate-sandwich made of a functionally graded (FG) core and FG face sheets, using FEM. The outcomes of the FEM analysis are confirmed, compared to those found in the literature. Furthermore, the effects of the major factors such as thermal load, geometrical factors, and volume fraction distribution on the thermo-mechanical bending behavior are assessed. This analysis provides useful information for the design and optimization of FG plates under thermal and mechanical loads. The key conclusions may be summed up as follows:

- The increase in temperature generates the amplification of the displacement.
- The normal stress decreases as the coefficient p increases, while it increases as the temperature increases.
- The increase in displacement with increasing a/h is only for relatively small a/h. At high a/h, the displacement even decreases slightly with increasing a/h.
- The influence of the index p is insignificant on the displacement.

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