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GENERAL MECHANICS

REDUCING ENERGY LOSSES IN SOME CLASSES OF WOODWORKING MACHINES – OPTIMIZATION SOLUTIONS

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ABSTRACT: In this paper, the losses of kinetic energy in big band saw machines are examined. Expressions have been obtained to calculate the kinetic energy of the mechanical system in the ideal and the real case. With the help of these expressions, the final dependencies for determining the energy losses for the studied class of machines were obtained. These dependences show the influence of linear and angular inaccuracies, i.e. of the parameters e and α. A number of optimization solutions have been proposed that allow the values of both parameters to be calculated so that energy losses are minimal. The proposed approach can be used in the design of other classes of woodworking machines, as well as in the study of energy losses for this class of machines.

KEY WORDS: woodworking machines, kinetic energy losses, optimization solutions.

1 INTRODUCTION

Band saw machines with big diameters of the leading wheels are used in woodworking industry for sawing various workpieces. These wheels are produced with corresponding linear and angular inaccuracies as a result of their big dimensions. In this case, the losses of kinetic energy of the band saw machines are very large in operating mode.

Scientific studies related to energy efficiency of different classes of woodworking machinery are published in the technical literature [1–5].

The purpose of the work presented in this paper is the determination of the kinetic energy losses of the band saw machines as a result of these inaccuracies. In order to fulfill this goal, it is necessary to determine the kinetic energy of the mechanical system for the two basic cases:

- determining the kinetic energy of the mechanical system without linear and angular inaccuracies of the leading wheels;

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- determining the kinetic energy of the mechanical system with linear and angular inaccuracies of the leading wheels.

With the help of these expressions we can get an expression to calculate the kinetic energy loss of the mechanical system in the real case. This expression allows to be analyzed the real state of the concrete machine and to be given some recommendations for the parameters variation of the woodworking machines, so that the kinetic energy losses to be minimum in operating mode.

Another purpose of the work presented in this paper is the optimization of the kinetic energy losses of the mechanical system caused by the corresponding linear and angular inaccuracies. In this case, we can use appropriate optimization procedures. These procedures determine the optimal values of the system parameters, so that the losses of kinetic and, respectively, electrical energy have a minimum value. In this way, it can be guaranteed that woodworking machines will operate with minimal energy losses.

2 Expose

In this study, the scheme of the cutting mechanism of the band saw machine is used. The dynamic models in the absence and presence of linear and angular inaccuracies are also proposed. They are used to fulfill the purposes of the study, as well as to solve the recorded tasks.

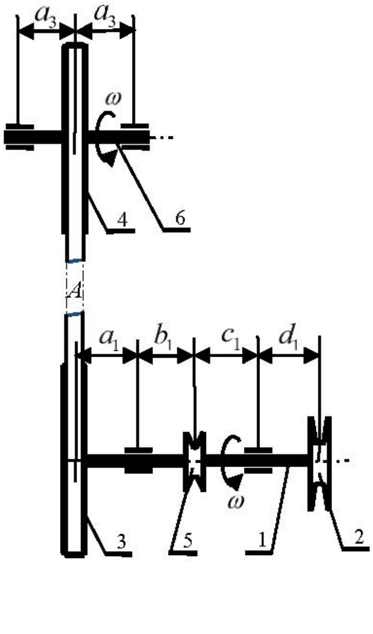


Fig. 1. Cutting mechanism. The following symbols are defined: 1 –main shaft; 2 and 5 – belt pulleys, 3 and 4 – leading wheels; A – band saw blade; and 6 – upper shaft.

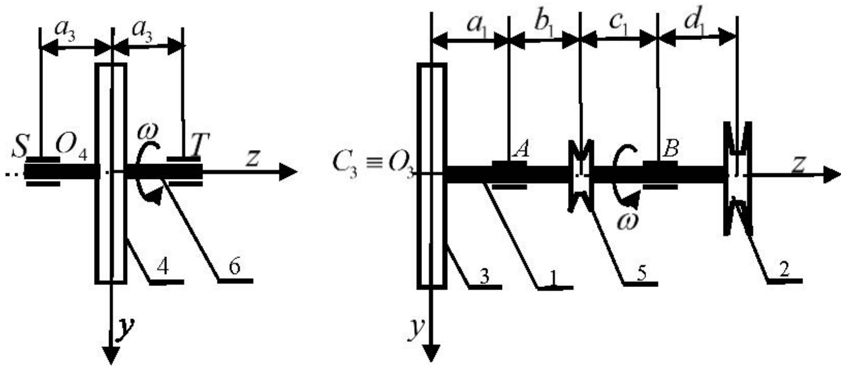
2.1 Scheme of the cutting mechanism

The leading wheel can be mounted on the upper shaft in two ways, which are described in other literary sources [6–9]. We consider the case, where the leading wheel is mounted between the two bearing supports.

The scheme of the cutting mechanism is shown in Fig. 1.

2.2 Kinetic energy of the mechanical system without linear and angular inaccuracies – ideal case

We use the dynamic models of the leading wheels mounted on the upper shaft and on the main shaft of the band saw machine. These models are shown in Figs. 2(a) and 2(b). The following fixed coordinate systems are used: coordinate system O_4xyz and coordinate system O_3xyz. In this case, the x axis points towards us. The shafts rotate at a constant angular velocity ω.



(a) (b)

Fig. 2. Dynamic models in the absence of linear and angular inaccuracies.

2.2.1 Kinetic energy of the mechanical system shown in Fig. 2(a) – ideal case

The kinetic energy of the mechanical system can be calculated from the following dependence [10–12]:

$$T_{pb} = \frac{1}{2} J_{pb} \omega^2 , \tag{1}$$

where J_{pb} is the mass moments of inertia of the mechanical system with respect to the axis of rotation,

$$J_{pb} = J_{4b} + J_{6b} , \tag{2}$$

where J_{4b} and J_{6b} are the mass moments of inertia of the leading wheel 4 and of the upper shaft 6 relative to the z axis.

2.2.2 Kinetic energy of the mechanical system shown in Fig. 2(b) – ideal case

The kinetic energy in this case can be calculated by the following expression.

$$T_p = \frac{1}{2} J_p \omega^2 \,, \tag{3}$$

where J_p is the mass moment of inertia of the system with respect to the axis of rotation, i.e. toward the axis z. We can calculate it from the following expression:

$$J_p = J_2 + J_3 + J_5 + J_1 \,. \tag{4}$$

In the above expression, J_2 and J_5 are the mass moments of inertia of the belt pulleys 2 and 5, J_3 is the mass moment of inertia of the leading wheel 3 and J_1 is the mass moment of inertia of the main shaft with respect to the axis of rotation. The angular velocity of the mechanical system toward the same axis is ω.

2.3 Kinetic energy of the mechanical system in the presence of linear and angular inaccuracies – real case

In this case, we use the dynamic models shown in Fig. 3. The leading wheels 3 and 4 are made with the corresponding geometric and angular inaccuracies. These inaccuracies are shown in the same figures and are denoted as geometric deviation $e = O_j C_j$, ($j = 3, 4$), where C_j are the centres of mass of the discs 3 and 4 and angular deviation α.

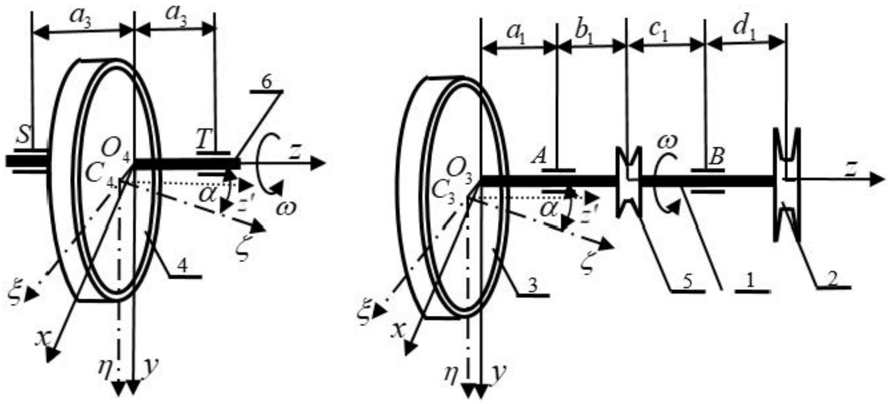
2.3.1 Kinetic energy of the mechanical system shown in Fig. 3(a) – real case

The dynamic model shown in Fig. 3(a) includes the upper leading wheel 4 and the upper shaft 6. We use the expression for calculating the kinetic energy of the mechanical system in its final form. This expression was obtained by the author in his previous work [8]

$$T_{rb} = \frac{1}{2} \left(J_{rb} + m_4 Z_{41}^2 \right) \omega^2 \,, \tag{5}$$

where J_{rb} is the mass moment of inertia of the mechanical system and can be calculated from the following expression:

$$J_{rb} = J'_{4b} + J_{6b} \,. \tag{6}$$



(a) (b)

Fig. 3. Dynamic models in the presence of linear and angular inaccuracies.

In the above expression, J'_{4b} is the mass moment of inertia of the leading wheel relative to the axis of rotation. This moment can be calculated from a known formula in the technical literature [10]

$$J'_{4b} = J_\zeta \cos^2\alpha + J_\xi \sin^2\alpha + m_4 e^2 \cos^2\alpha\,, \tag{7}$$

where J_ζ and J_ξ are the mass moments of inertia of the leading wheel with respect to the principal axes of inertia ζ and ξ, m_4 is the mass of leading wheel 4.

The function Z_{41} is calculated from the expression (8)

$$Z_{41} = \overline{S}_1 \cos\omega_0 a_3 + \overline{T}_1 \sin\omega_0 a_3 + \overline{V}_1 ch\omega_0 a_3 + \overline{W}_1 \sinh\omega_0 a_3\,. \tag{8}$$

In the expressions (8) ω_0 is parameter, $(\omega_0^4 = A_{ms}\rho\omega^2/EJ)$ [13, 14], where E is the modulus of elasticity, $J = J_x = J_y$ is the axial moment of inertia, A_{ms} is the cross-sectional area of the shaft, ρ is the density of the material. $\overline{S}_1, \overline{T}_1, \overline{V}_1, \overline{W}_1$ are the constants of integration. They can be calculated from dependencies (9)

$$\overline{S}_1 = \frac{\Delta_{\overline{S}_1}}{\Delta_P}\,, \qquad \overline{T}_1 = \frac{\Delta_{\overline{T}_1}}{\Delta_P}\,, \qquad \overline{V}_1 = \frac{\Delta_{\overline{V}_1}}{\Delta_P}\,, \qquad \overline{W}_1 = \frac{\Delta_{\overline{W}_1}}{\Delta_P}\,, \tag{9}$$

where $\Delta_P = \det P$ is the determinant of the matrix **P**. This determinant is non-zero, because otherwise the mechanical system will operate in resonance mode

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \cos\omega_0 a_3 & \sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 \\ -\sin\omega_0 a_3 & \cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 \\ 0 & 0 & 0 & 0 & \cos 2\omega_0 a_3 & \sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\cos 2\omega_0 a_3 & -\sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\sin 2\omega_0 a_3 & \cos 2\omega_0 a_3 & -\sinh 2\omega_0 a_3 & -\cosh 2\omega_0 a_3 \end{bmatrix} \tag{10}$$

The determinants $\Delta_{\overline{S}_1}$, $\Delta_{\overline{T}_1}$, $\Delta_{\overline{V}_1}$ and $\Delta_{\overline{W}_1}$ are determinants of the matrices formed by the matrix **P**, in which the corresponding columns are replaced by the column vector **p** as follows:

$$\begin{aligned} &\Delta_{\overline{S}_1} - \text{the first column of the matrix } \mathbf{P}, \\ &\Delta_{\overline{T}_1} - \text{the second column of the matrix} \mathbf{P}, \\ &\Delta_{\overline{V}_1} - \text{the third column of the matrix } \mathbf{P}, \\ &\Delta_{\overline{W}_1} - \text{the fourth column of the matrix } \mathbf{P}. \end{aligned} \tag{11}$$

The column vector **p** has the following form:

$$\mathbf{p} = \begin{bmatrix} 0 & \dfrac{S_{bm}}{EJ\omega_0^3} & 0 & 0 & 0 & 0 & 0 & \dfrac{T_{bm}}{EJ\omega_0^3} \end{bmatrix}^T. \tag{12}$$

For example, the determinant $\Delta_{\overline{S}_1}$ is written in the following way:

$$\Delta_{\overline{S}_1} = \tag{13}$$
$$\begin{vmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{S_{bm}}{EJ\omega_0^3} & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 \\ 0 & \cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 \\ 0 & 0 & 0 & 0 & \cos 2\omega_0 a_3 & \sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\cos 2\omega_0 a_3 & -\sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ \frac{T_{bm}}{EJ\omega_0^3} & 0 & 0 & 0 & -\sin 2\omega_0 a_3 & \cos 2\omega_0 a_3 & -\sinh 2\omega_0 a_3 & -\cosh 2\omega_0 a_3 \end{vmatrix}$$

The amplitudes S_{bm} and T_{bm} are calculated from the following dependencies:

$$\begin{aligned} S_{bm} &= -\frac{1}{4a_3}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big)e\cos\alpha \\ &\quad + \frac{\omega^2}{2}\Big[\frac{1}{a_3}(J_\xi - J_\zeta - m_4 e^2)\sin\alpha - m_4 e\Big]\cos\alpha, \\ T_{bm} &= \frac{1}{4a_3}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big)e\cos\alpha \\ &\quad - \frac{\omega^2}{2}\Big[\frac{1}{a_3}(J_\xi - J_\zeta - m_4 e^2)\sin\alpha + m_4 e\Big]\cos\alpha. \end{aligned} \tag{14}$$

In the above dependencies $R_\Sigma$is the total resistance force between the workpiece and the band saw machine. H is the thickness of the workpiece, $V$is the cutting speed, u is the feeding speed, b is the width of the cutter, m is a coefficient, $0 \le m \le 1$. $K_{\Delta(\lambda)}$ is the specific work of the cutting. It is determined from the expressions known in technical literature [6, 15, 16].

2.3.2 Kinetic energy of the mechanical system shown in Fig. 3(b) – real case

The dynamic model shown in Fig. 3(b) includes the lower leading wheel 3, the main shaft 1 and the belt pulleys 2 and 5. In this case, we use the following expression to calculate the kinetic energy of the mechanical system [8]

$$T_r = \frac{1}{2}\left(J_r + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right)\omega^2. \tag{15}$$

In this expression m_3, m_5 and m_2 are the masses of the three disks. J_r is the mass moment of inertia of the system in the presence of linear and angular inaccuracies, i.e., in the real case. This moment can be calculated from the following expression:

$$J_r = J_2 + J_3^{'} + J_5 + J_1\,. \tag{16}$$

The mass moment of inertia of the leading wheel with respect to the axis of rotation is denoted by J_3' and can be calculated using expression (17)

$$J_3' = J_\zeta \cos^2\alpha + J_\xi \sin^2\alpha + m_3 e^2 \cos^2\alpha\,, \tag{17}$$

where J_ζ and J_ξ are the mass moments of inertia of the leading wheel 3 with respect to the principal axes of inertia ζ and ξ.

The functions Z_{2i}, $(i = 1 \div 3)$ are calculated from the dependencies written below

$$\begin{aligned} Z_{21} &= \left(\overline{R}_1 + \overline{V}_1\right)\,, \\ Z_{22} &= \left(\overline{R}_2 \cos\omega_0 z_2 + \overline{S}_2 \sin\omega_0 z_2 + \overline{V}_2 ch\omega_0 z_2 + \overline{W}_2 sh\omega_0 z_2\right)\,, \\ Z_{23} &= \left(\overline{R}_3 \cos\omega_0 z_3 + \overline{S}_3 \sin\omega_0 z_3 + \overline{V}_3 ch\omega_0 z_3 + \overline{W}_3 sh\omega_0 z_3\right)\,, \end{aligned} \tag{18}$$

where the coordinates z_i $(i = 1 \div 3)$ have the following values: $z_1 = 0$, $z_2 = a_1 + b_1$, $z_3 = a_1 + b_1 + c_1 + d_1 = l$. $\overline{R}_i$, $\overline{S}_i$, $\overline{V}_i$, $\overline{W}_i$ are the constants of integration. They can be calculated from the following dependencies:

$$\overline{R}_i = \frac{\Delta_{\overline{R}_i}}{\Delta_B}\,,\quad \overline{S}_i = \frac{\Delta_{\overline{S}_i}}{\Delta_B}\,,\quad \overline{V}_i = \frac{\Delta_{\overline{V}_i}}{\Delta_B}\,,\quad \overline{W}_i = \frac{\Delta_{\overline{W}_i}}{\Delta_B},\qquad (i = 1,\, 2\, 3)\,, \tag{19}$$

where $\Delta_B = \det B$ is the determinant of the matrix $\mathbf{B}$, which has the following form [8, 15]:

(20) $\mathbf{B} =$

columns 1 through 4

$$\begin{matrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ \cos\omega_0 a_1 & \sin\omega_0 a_1 & \cosh\omega_0 a_1 & \sinh\omega_0 a_1 \\ -\sin\omega_0 a_1 & \cos\omega_0 a_1 & \sinh\omega_0 a_1 & \cosh\omega_0 a_1 \\ -\sin\omega_0 a_1 & \cos\omega_0 a_1 & -\sinh\omega_0 a_1 & -\cosh\omega_0 a_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

columns 5 through 8

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin\omega_0 a_1 & -\cos\omega_0 a_1 & -\sinh\omega_0 a_1 & -\cosh\omega_0 a_1 \\ \sin\omega_0 a_1 & -\cos\omega_0 a_1 & \sinh\omega_0 a_1 & \cosh\omega_0 a_1 \\ \cos\omega_0 a_1 & \sin\omega_0 a_1 & \cosh\omega_0 a_1 & \sinh\omega_0 a_1 \\ \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) \\ -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & -\sinh\omega_0(a_1+b_1+c_1) & -\cosh\omega_0(a_1+b_1+c_1) \\ -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

columns 9 through 12

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sin\omega_0(a_1+b_1+c_1) & -\cos\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) \\ \sin\omega_0(a_1+b_1+c_1) & -\cos\omega_0(a_1+b_1+c_1) & -\sinh\omega_0(a_1+b_1+c_1) & -\cosh\omega_0(a_1+b_1+c_1) \\ \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & \cosh\omega_0(a_1+b_1+c_1) & \sinh\omega_0(a_1+b_1+c_1) \\ -\cos\omega_0(a_1+b_1+c_1+d_1) & -\sin\omega_0(a_1+b_1+c_1+d_1) & \cosh\omega_0(a_1+b_1+c_1+d_1) & \sinh\omega_0(a_1+b_1+c_1+d_1) \\ \sin\omega_0(a_1+b_1+c_1+d_1) & -\cos\omega_0(a_1+b_1+c_1+d_1) & \sinh\omega_0(a_1+b_1+c_1+d_1) & \cosh\omega_0(a_1+b_1+c_1+d_1) \end{matrix}$$

The other determinants in (19) are determinants of matrices that are formed by matrix **B**. In this matrix, certain columns are replaced by the column-vector **b** as follows:

(21)
$$\begin{aligned} &\Delta_{\overline{R}_i}\ (i = 1 \div 3) - \text{the first, the fifth, or the ninth column,} \\ &\Delta_{\overline{S}_i}\ (i = 1 \div 3) - \text{the second, the sixth or the tenth column,} \\ &\Delta_{\overline{V}_i}\ (i = 1 \div 3) - \text{the third, the seventh or the eleventh column,} \\ &\Delta_{\overline{W}_i}\ (i = 1 \div 3) - \text{the fourth, the eighth or the twelfth column.} \end{aligned}$$

The column-vector **b** is written below,

(22)
$$\mathbf{b} = \left[\frac{m_3 e\omega^2 \cos\alpha}{EJ\omega_0^3}\ \frac{M_{bm}}{EJ\omega_0^2}\ 0\ 0\ \frac{A_{bm}}{EJ\omega_0^3}\ 0\ 0\ \frac{B_{bm}}{EJ\omega_0^3}\ 0\ 0\ 0\ 0\right]^T.$$

The symbols E, J and are explained in formula (8). A_{bm} , B_{bm} and M_{bm} can be calculated from the following expressions:

$$
\begin{aligned}
A_{bm} &= \frac{1}{2(b_1+c_1)}\Big[\omega^2(J_\xi - J_\varsigma)\sin 2\alpha \\
&\quad - 2\omega^2 m_3(a_1+b_1+c_1+e\sin\alpha)e\cos\alpha - (R_b^n + R_\Sigma)e\cos\alpha\Big]\,, \\
B_{bm} &= \frac{1}{2(b_1+c_1)}\Big[\,(R_b^n + R_\Sigma)\,e\cos\alpha \\
&\quad + 2\omega^2 m_3(a_1+e\sin\alpha)e\cos\alpha - \omega^2(J_\xi - J_\varsigma)\sin 2\alpha\Big]\,, \\
M_{bm} &= \frac{1}{2}\left[(R_b^n + R_\Sigma)\,e\cos\alpha - \omega^2\left(J_\xi - J_\varsigma - m_3 e^2\right)\sin 2\alpha\right]\,,
\end{aligned}
\tag{23}
$$

where R_b^n is the normal force loading the workpiece. This force can be calculated from expressions that are used in various sources [6,15,16]. R_Σ is the total resistance force between the workpiece and the band saw machine. This force is calculated for each individual case.

2.4 Loss of kinetic energy of the mechanical system

2.4.1 Loss of kinetic energy of the mechanical system consisting of the upper shaft and the upper leading wheel

In this case, the loss of kinetic energy is calculated from the difference of kinetic energies in the real and ideal cases.

$$
{}_\triangle T_b = T_{rb} - T_{pb}\,. \tag{24}
$$

After replacing T_{rb} and T_{pb} with their equals, the following expression is obtained:

$$
{}_\triangle T_b = \frac{1}{2}\left(J_{rb} + m_4 Z_{41}^2\right)\omega^2 - \frac{1}{2}J_{pb}\omega^2\,. \tag{25}
$$

From this expression, we obtain the expression for loss of kinetic energy in final form

$$
\begin{aligned}
{}_\triangle T_b = \frac{1}{2}\Bigg[&(J_\xi - J_\zeta)\sin^2\alpha + m_4 e^2\cos^2\alpha \\
&+ m_4(\overline{S}_1\cos\omega_0 a_3 + \overline{T}_1\sin\omega_0 a_3 + \overline{V}_1 ch\omega_0 a_3 + \overline{W}_1 sh\omega_0 a_3)^2\Bigg]\omega^2.
\end{aligned}
\tag{26}
$$

2.4.2 Loss of kinetic energy of the mechanical system consisting of the main shaft, the lower leading wheel and the belt pulleys

The loss of kinetic energy can be calculated from the following expression:

$$\triangle T = T_r - T_p \,. \tag{27}$$

We replace T_r and T_p with their equal expressions written in (15) and (3) and get the following expression:

$$\triangle T = \frac{1}{2}(J_r + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2)\omega^2 - \frac{1}{2} J_p \omega^2 \,. \tag{28}$$

After converting the above expression, we obtain the following dependence for calculating the kinetic energy loss of the mechanical system.

$$\triangle T = \frac{1}{2}\left[(J_\xi - J_\zeta)\sin^2\alpha + m_3 e^2 \cos^2\alpha + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right]\omega^2. \tag{29}$$

2.4.3 Complete loss of kinetic energy of the mechanical system

The complete kinetic energy loss of the mechanical system is the sum of the energy loss of the upper shaft and upper leading wheel and the energy loss of the main shaft, lower leading wheel, and two belt pulleys. This loss can be calculated from the following expression:

$$\triangle T_\Sigma = \triangle T_b + \triangle T \,. \tag{30}$$

We replace $\triangle T_b$ and $\triangle T$ with their equals and after appropriate transformations we get the expression for the complete loss of kinetic energy in its final form.

$$\begin{aligned}\triangle T_\Sigma = \frac{1}{2}\Bigg[&(J_\xi - J_\zeta)\sin^2\alpha + m_4 e^2\cos^2\alpha \\ &+ m_4(\overline{S}_1\cos\omega_0 a_3 + \overline{T}_1 \sin\omega_0 a_3 + \overline{V}_1 ch\omega_0 a_3 + \overline{W}_1 sh\omega_0 a_3)^2 \\ &+ (J_\xi - J_\zeta)\sin^2\alpha + m_3 e^2\cos^2\alpha + m_3(\overline{R}_1 + \overline{V}_1)^2 \\ &+ m_5(\overline{R}_2\cos\omega_0 z_2 + \overline{S}_2\sin\omega_0 z_2 + \overline{V}_2 ch\omega_0 z_2 + \overline{W}_2 sh\omega_0 z_2)^2 \\ &+ m_2(\overline{R}_3\cos\omega_0 z_3 + \overline{S}_3\sin\omega_0 z_3 + \overline{V}_3 ch\omega_0 z_3 + \overline{W}_3 sh\omega_0 z_3)^2\Bigg]\omega^2 \,.\end{aligned} \tag{31}$$

This expression allows to calculate the loss of kinetic energy at different values of the linear and angular inaccuracies, as well as different values of the masses and of the geometric and kinematic characteristics of the mechanical system.

2.5 Kinetic energy losses in big band saw machines – optimization

2.5.1 Optimization solutions reducing energy losses

In this case, the loss of kinetic energy depends on the change of many parameters such as the presence of linear and angular deviations, the change of linear and angular velocities, mass moments of inertia, axial moments of inertia, etc. This can be seen from the expression (31), as well as from the dependencies for the constants of integration involved in this expression. The purpose of this part of the study is to propose optimization solutions that minimize kinetic energy losses. To achieve this purpose, we use the optimization procedure *fmincon* [17, 18]. This procedure looks for the minimum of a multidimensional function with the corresponding constraints. In this case, the interval of constraints in which the parameters e and α change must be determined. These constraints are expressed by the following inequalities: $lb \leq [e, \alpha] \leq ub$, where lb and ub are the lower and upper bounds. It is also necessary to determine the initial approximation $x0$ around which the function has a local minimum. The way in which the optimization procedure is applied is shown below.

```
function F_def
x0=[Initial Approximation];
lb=[Lower Bounds];
ub=[Upper Bounds];
ops=optimset('LargeScale','off');
[x,fval,exitflag,output]=fmincon(@def1,x0,A,b,Aeq,beq,lb,ub,nonlcon,ops),
function F1=def1(x)
.
objective function
.
F1 =[see expressions for calculating the loss of kinetic energy]
.
```

2.5.2 Optimization procedure – application

The leading wheels, both for the upper shaft and for the main shaft of the big band saw machines, are made with large geometric dimensions. The diameters of these wheels can reach up to 3200 mm. For this reason, they are always made with inaccuracies. The magnitudes of these inaccuracies depend on many factors, such as different manufacturing technologies, different assembly technologies, different operating conditions, and more. For this reason, we use the allowable technological boundaries in which the parameters e and α are changed, i.e. $e_{\min} \leq e \leq e_{\max}$, $\alpha_{\min} \leq \alpha \leq \alpha_{\max}$. These boundaries determine the zone in which the band saw machine can operate. This zone is called the *working zone*. The area under the working zone is called the *initial zone* and the area above the working zone is called the *final* *zone*. We can apply the optimization procedures for the three zones. However, this is not always necessary. The most desirable area for the leading wheels to operate is the initial zone. The most undesirable area for the leading wheels to operate is the final zone. The most important area for which we are looking for an optimization solution is the working zone. We can obtain the optimal values of the parameters e and α so that the loss of kinetic energy has a minimum value.

We can calculate the values for the change of kinetic energy using the obtained expression (31). In this case, the parameter e changes from 0 to 0.0015 with step 0.0003 and the parameter α changes from 0 to 0.026175 with step 0.001745. The obtained values are presented below as a matrix.

$$\triangle T_{\Sigma} = 1.0e+$$

$$+003 \begin{bmatrix} 0.0000 & 0.0146 & 0.0585 & 0.1316 & 0.2339 & 0.3654 & 0.4631 & 0.7161 \\ 0.0032 & 0.0283 & 0.0827 & 0.1664 & 0.2792 & 0.4213 & 0.5262 & 0.7931 \\ 0.0126 & 0.0483 & 0.1133 & 0.2075 & 0.3309 & 0.4835 & 0.5926 & 0.8764 \\ 0.0284 & 0.0747 & 0.1502 & 0.2549 & 0.3888 & 0.5520 & 0.6654 & 0.9660 \\ 0.0505 & 0.1073 & 0.1933 & 0.3086 & 0.4531 & 0.6268 & 0.7444 & 1.0619 \\ 0.0789 & 0.1462 & 0.2428 & 0.3686 & 0.5237 & 0.7080 & 0.8298 & 1.1641 \\ 0.9353 & 1.1837 & 1.4612 & 1.7680 & 2.1038 & 2.4689 & 0.9215 & 3.2863 \\ 1.0228 & 1.2817 & 1.5698 & 1.8871 & 2.2335 & 2.6091 & 2.8630 & 3.4475 \\ 1.1167 & 1.3861 & 1.6848 & 2.0125 & 2.3695 & 2.7556 & 3.1708 & 3.6151 \\ 1.2168 & 1.4968 & 1.8060 & 2.1443 & 2.5118 & 2.9084 & 3.3341 & 3.7890 \\ 1.3233 & 1.6138 & 1.9335 & 2.2824 & 2.6604 & 3.0675 & 3.5038 & 3.9692 \\ 1.4360 & 1.7371 & 2.0674 & 2.4268 & 2.8153 & 3.2330 & 3.6798 & 4.1557 \end{bmatrix}$$

The values at which the kinetic energy loss changes for this case varies from zero to 4155.7 [J]. To find optimization solutions for the three zones, we use the optimization procedure namely *fmincon*. And in this case, it is necessary to determine in advance the initial approximation x0, lower bounds lb, upper bounds ub, as well as to compile the objective function for the three zones.

INITIAL ZONE

```
function F_def
    x0=[0.00075 0.0026175];
    lb=[0.0003 0];
    ub=[0.0012 0.005235];
    ops=optimset('LargeScale','off');
    [x,fval,exitflag,output]=fmincon(@def1,x0,[],[],[],[],lb,ub,[],ops)
    function F1=def1(x)
    .
    objective function
    .
    F1 =[see expressions for calculating the loss of kinetic energy (31)]
```

The following results are obtained after startup of the procedure.

$e = $ 1.0e - 003*0.3000 [m], $\alpha = 0$, ${}_{\Delta}T_{\Sigma\,\min} = 3.154387$ [J].
exitflag = 1
output =
algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'
Ideal case: $e = 0$, $\alpha = 0$, ${}_{\Delta}T_{\Sigma\,\min} = 0$.

WORKING ZONE

```
function F_def
    x0 = [0.00075 0.0130875];
    lb = [0.0003 0.005235];
    ub = [0.0012 0.02094];
    ops = optimset('LargeScale','off');
    [x,fval,exitflag,output] = fmincon(@def1,x0,[],[],[],[],lb,ub,[],ops)
    .
    function F1 = def1(x)
    .
```

objective function

.

$F_1 =$ [see expressions for calculating the loss of kinetic energy (31)]

.

The following results are obtained after startup of the procedure.

$e = 0.0003$ [m], $\alpha = 0.0052$ [rad], ${}_{\Delta}T_{\Sigma\,\min} = 166.3548$ [J].
exitflag = 1
output =
algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'

FINAL ZONE

```
function F_def
    x0 = [0.00075 0.023558];
    lb = [0.0003 0.02094];
    ub = [0.0012 0.026175];
    ops = optimset('LargeScale','off');
    [x,fval,exitflag,output] = fmincon(@def1,x0,[],[],[],[],lb,ub,[],ops)
    function F1 = def1(x)
    .
```

objective function

.

F_1 = [see expressions for calculating the loss of kinetic energy (31)]

.

The following results are obtained after startup of the procedure.

e = 1.0e - 003*0.3000 [m], $\alpha = 0.0209$ [rad], $_\triangle T_{\Sigma\min} = 2.2335e + 003$ [J]
exitflag =1
output =
algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'

THE WORST CASE:

$$e = 0.0015 \text{ [m]}, \quad \alpha = 0.026175 \text{ [rad]}, \quad {}_\triangle T_{\Sigma\min} = 4.1557e + 003 \text{ [J]}.$$

In this part of the paper, optimization solutions are proposed for determining the minimum value of kinetic energy loss $_\triangle T_{\Sigma_{\min}}$. As can be seen from the numerical values written as matrices, the loss of kinetic energy depends on the two parameters e and α and changes in the following limits $0 \leq {}_\triangle T_\Sigma \leq 4155.7$ [J].

The most important area in which the band saw machine works is the working zone. The minimum energy loss for this zone is $_\triangle T_{\Sigma_{\min}} = 166.3548$ [J] The values of the two parameters at which the loss of kinetic energy assumes minimum values were also obtained, which are $e = 0.0003$ [m]. $\alpha = 0.0052$ [rad]. These values can be used in the design of new band saw machines in order to guarantee minimum energy consumption.

The initial zone is the most desirable zone in which the band saw machine can operate. Theoretically, the energy loss in this zone can be zero. This can be obtained when there are no linear and angular deviations, i.e. $e = 0$, $\alpha = 0$, $_\triangle T_{\Sigma_{\min}} = 0$. Unfortunately, this is technologically impossible. The actual value calculated by the optimization procedures is $_\triangle T_{\Sigma_{\min}}$= 3.154387 [J] when $\alpha = 0$, e = 1.0e-003*0.3000 [m].

The most undesirable zone in which the band saw machine can operate is the final zone. In this zone, the loss of kinetic energy assumes maximum value, which is $_\triangle T_{\Sigma_{\min}}$ = 4.1557e+003 [J]. This value is calculated for the maximum values of the two parameters, $e = 0.0015$ [m], $\alpha = 0.026175$ [rad]. The actual value calculated from the optimization procedure is $_\triangle T_{\Sigma_{\min}}$ = 2.2335e+003 [J]. If the band saw machine works in this area, technological solutions should be applied, the sizes of linear and angular inaccuracies should be reduced and the machine should work in the working zone.

3 Conclusion

In this paper, the kinetic energy losses that occur in the operating mode in big band saw machines are investigated. These losses are caused by the linear and angular inaccuracies with which the leading wheels are manufactured. For this purpose, expressions are derived to determine the kinetic energy. The kinetic energy without linear and angular inaccuracies has a constant value when the shafts are rotate with constant angular velocity. The kinetic energy in case of linear and angular inaccuracies has a variable value that depends on different factors. Through these expressions dependence is derived that allows the loss of the kinetic energy to be calculated. We can establish the fact that the loss of the kinetic energy depends on the unbalance e and the angle α. This loss will be equal to zero when the eccentricity e and the angle α are equal to zero. In fact, they are different from zero and the more their values are bigger the more the kinetic energy loss is bigger. On the other hand, the loss of the kinetic energy of the band saw machine depends on the masses and the diameters of the leading wheels. This loss also depends similarly on the lengths and the cross-sections of the main and upper shafts. We can establish the cases when the energy loss will be equal to zero or this loss will have a minimum value. Solving of the optimization task allows the parameters of the machines to be chosen in such a way that the losses of the kinetic energy to be minimum.

In conclusion, this research can be seen as a contribution to the contemporary knowledge on energy conservation and design of energy-saving machines. It may contribute to better understanding of these problems and prove useful for many researchers. In particular, the proposed approach can be used in the design of other classes of woodworking machines, as well as in the study of energy losses for this class of machines.

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